UNIVERSITY OF PENNSYLVANIA

An Investigation of Certain Abilities Fundamental to the Study of Geometry

BY

JOHN HARRISON MINNICK

A THESIS

PRESENTED TO THE FACULTY OF THE GRADUATE SCHOOL IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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ACKNOWLEDGMENTS

I wish to acknowledge my indebtedness to those whose aid has made this study possible. Dean H. L. Smith of Indiana University, Mr. E. E. Arnold of Albany, New York, and many superintendents, principals and teachers throughout the country gave valuable assistance in gathering data. Doctors Ralph and Robert Duncan of the University of Pennsylvania, Mr. Roy Cumins of the Philadelphia Public Schools and Miss Jessie A. Smith of Indiana University read the manuscript and offered many useful suggestions. Especially am I indebted to Professor A. Duncan Yocum for valuable assistance given throughout the entire investigation. I wish also to acknowledge my indebtedness to my wife, without whose help with the many details of the work this study would have been impossible.

J. H. M.

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INTRODUCTION

In 1911–12 the author had charge of about forty pupils in second-term plane geometry in the Bloomington, Indiana, high school. Most of these pupils came from good homes and apparently should have been able to do satisfactory work. Such, however, was not the case. Crudely constructed tests revealed almost complete lack of certain abilities believed to be fundamental to the study of geometry. Special attention given to these abilities resulted in a decided improvement in the work of the group. Since then the author has experimented along these same lines with other classes and in the light of this experience it seemed desirable to determine the relation of these specific abilities to general geometrical ability. During the years 1915–17 he conducted an extensive investigation which is reported in the following pages.

So far as the author knows, no similar investigation has been carried on in this field. L. V. Stockard and J. Carlton Bell[1] have made "A Preliminary Study of the Measurement of Abilities in Geometry," but it does not cover the ground of this investigation. Suggestions as to methods of procedure and of handling the data have been gathered from the following: Thorndike's "Mental and Social Measurements," Brown's "Mental Measurement," King's "Elements of Statistical Methods," Buckingham's "Spelling Ability—Its Measurement and Distribution," Stone's "Arithmetical Abilities, Some Factors Determining Them," Trabue's "Completion-Test Language Scales," and Woody's "Measurements of Some Achievements in Arithmetic."

This report consists of two parts. The first is a brief synopsis of the method and results and is intended for those who care only for the conclusions; the second is a more detailed statement including the data, and is intended for those who care to investigate the study more carefully.

[1] *Journal of Educational Psychology*, Vol. 7, pp. 567–580.

AN INVESTIGATION OF CERTAIN ABILITIES FUNDAMENTAL TO THE STUDY OF GEOMETRY

PART I

Purpose of the Investigation.—Success in the formal demonstration of a theorem of geometry is dependent upon at least four abilities;[1] namely,

1. The ability to draw a figure for the theorem,
2. The ability to state concretely and accurately the hypothesis and conclusion of the theorem,
3. The ability to recall additional facts about a figure when one or more facts are given, and
4. The ability to select from the available facts those that are necessary for a proof and to arrange them so as to arrive at the desired conclusion.

The purpose of this investigation is threefold:

1. To determine the relation of each of these four abilities to the teachers' marks. This should, in turn, determine either the extent to which teachers value these abilities or the degree to which they are able to base their marks on the things which they do value.
2. To determine the extent to which these abilities are developed in our high schools.
3. To develop tests which may be used for the purpose of diagnosis; that is for the purpose of determining whether or not the weaknesses of a class are due to the lack of development of one or more of these abilities.

Plan of Procedure.—For these purposes four tests[2] (to be known as A, B, C, and D) have been developed, each testing one of the abilities in question. A fifth test (E) requiring pupils to draw the auxiliary lines for exercises was developed to supple-

[1] Pages 6–7.

[2] Pages 10–19.

ment those testing abilities 3 and 4 above. In order to avoid the effect of extensive class drill, original exercises were selected for each of these tests. None of the exercises involved concepts or knowledge beyond the first two books of geometry.

Each test was given to at least one thousand pupils, and after the papers were carefully marked the coefficient of correlation between the test scores and the school grades was computed for each school tested. These coefficients serve as indices of the relation between the pupils' abilities and the teachers' marks. The median scores for each test have been determined and they will serve as standards for determining whether or not a class is weak in respect to any one of these abilities.

Giving the Tests.—The tests were given in sixty-three schools in the states of New York, Rhode Island, New Jersey, Pennsylvania, West Virginia, Ohio, Indiana, Illinois, Iowa and Minnesota. Each test was given to at least one thousand pupils, and in all 5,195 pupils were tested. Among these was included almost every type of pupil. The tests were given when the classes had completed the first two books of geometry, thus avoiding the extensive elimination of pupils which occurs later in the course. This procedure is further justified by the fact that the abilities with which this study is concerned should be developed to a considerable degree by this time, and for the purpose of diagnosis it is important that the tests should be given as early as possible. The wide distribution of schools made it impossible for the author to conduct the tests in person. Therefore, the teacher of each class tested her own pupils. To insure uniformity a simple but definite set of directions was sent to each examiner.[1]

Scoring the Papers.—Two scores were kept for each pupil. The one, to be known as the *positive* score, is based on the per cent. of necessary statements correctly given; and the other, to be known as the *negative* score, is the total number of incorrect and unnecessary statements in his paper. We know of no way of combining these two elements; and as there is, by no means, a perfect correlation[2] between the two scores neither can be neglected on the ground that the other gives a perfect

[1] Pages 21–24.

[2] Pages 25–26.

representation of the pupil's ability. Furthermore, for the purpose of diagnosis it is important that we have the analytic view given by the separate scores rather than that given by a combination of the results. Hence for our purpose it seems best to keep the two scores separately.

In order to secure uniformity the author marked all papers. Before beginning to mark the papers each exercise of a test was carefully solved and the number of necessary steps noted as a basis for computing the positive scores. Each exercise of a test was then scored separately for an entire school; and if any answer gave particular difficulty, a record of it was kept for reference in all similar cases. After the papers of all pupils taking a given test were thus marked the exercises of that test were weighted according to the average positive scores.[1] A pupil's final positive score was obtained by marking each exercise on the basis of the weighted value thus assigned and then adding these marks for all exercises of the test.

Due to the great variation in the nature of the incorrect and unnecessary statements it was not possible to weight each error separately, and as there was no upper limit to the number of such statements that could be made we had no basis for comparing the negative difficulties of two exercises.[2] Hence the exercises were not weighted according to the negative scores. The total number of incorrect and unnecessary statements in a pupil's paper was taken as his negative score.

Criticism of the Tests.—If the time available for giving the tests had permitted a larger number of exercises to be used, more satisfactory results would, no doubt, have been obtained. An examination of the data[3] shows that the interval of difficulty covered by the exercises of each test is too small and that the exercises are not distributed uniformly throughout that interval. In general, the distribution curves[4] given by either the positive or negative scores are skewed toward the high end of the scale, but this skewness would, perhaps, largely disappear if the two scores could be combined. Furthermore, since we are concerned chiefly with the ranks of pupils when arrayed according to their

[1] Pages 43–49.
[2] Page 44.
[3] Pages 51–52.
[4] Pages 53–58.

abilities, it is not so important that the test show the exact difference between the abilities of two pupils as it is that they show which of the two is the better. That Tests A, B, C and D satisfy this condition is shown by the fact that there is but a slight tendency to group the pupils about a few points of the scale. However, in Test E the pupils are for the most part grouped at only four points.[1] Hence we may conclude that for our purpose Tests A, B, C and D are fairly satisfactory but that Test E is not.

Criticism of Teachers' Grades.—An examination of the grades given by the teachers to the 5,195 pupils tested shows that the grades of a large number of schools taken together give a fairly normal distribution curve.[2] If, however, the grades of the schools are considered separately, there is a great variation in the form of the distribution.[3] Occasionally the curve of a school is remarkably normal, but usually it presents some marked irregularity. Hence we may conclude that usually teachers' grades are not reliable measures of pupils' abilities.

Coefficients of Correlation.—The coefficient of correlation between the test and school grades has been computed for each school tested.[4] This coefficient varies from -0.150 to 0.697 for Test A; from 0.140 to 0.588 for Test B; from 0.042 to 0.548 for Test C; from 0.126 to 0.568 for Test D; and from 0.139 to 0.608 for Test E. In the case of each test the coefficient is usually small. If the abilities which this study investigates are of value in themselves, or if they form the basis for other results which are of value, they should bear a closer relation to the school grades than these coefficients indicate. If, on the other hand, the coefficients of correlation can be taken as indices of the values of these abilities, then these values are so slight that the schools are scarcely justified in giving as much time to this phase of geometry as is now given to it.

Standards of Achievements.—The median positive scores[5] for the different tests are: Test A, 62.5; Test B, 69.3; Test C, 50.6; Test D, 73.3; and Test E, 61.5. The median negative

[1] Tables XXVII–XXXVII.

[2] Page 59.

[3] Pages 61–63.

[4] Tables XVIII–XXII.

[5] Page 94.

scores are: Test A, 7.1; Test B, 3.5; Test C, 4.1; Test D, 2.6; and Test E, 2.5. A study of the data from each school shows that in the case of each test, the marks of some of the schools are quite satisfactory while those of others are extremely low. Local conditions have, no doubt, tended to lower the marks of some schools; nevertheless, it is difficult to see why the results should be so poor in some cases. If the abilities tested are essential to success in the study of geometry, then the results indicate that progress in some schools is almost impossible until these abilities have been further developed. On the other hand the achievements of other schools indicate that it is altogether possible to develop these abilities to a fair degree during the study of the first two books of geometry.

PART II

I. Purpose of the Investigation

High school geometry should include both the formal and the practical phases of the subject. The most important parts of formal geometry are the demonstration of theorems, the construction of figures under given conditions, and the solution of numerical problems. This study is limited to an investigation of certain fundamental abilities involved in the demonstration of theorems.

An examination of the steps in a demonstration will reveal these abilities. The first step in a demonstration is to draw the figure described in the theorem. As a second step, the pupil should state the hypothesis and conclusion accurately in terms of his figure. The third step is to recall additional known facts concerning the figure. Only in the simplest cases will the conclusion follow directly and solely from the facts stated in the hypothesis. Hence it is necessary for the pupil to have the important properties of a geometrical figure so definitely associated that he can recall them at will. Finally, as a fourth step, it is necessary to select from all the available facts those essential to the proof and to arrange them in the order necessary to arrive at the desired conclusion. Here there are really two steps involved,—the selection and the arrangement of facts. However, these steps are so closely related that it seems impossible to separate them for the purposes of this investigation. The selection of facts to be used will depend upon the arrangement or the method of proof. On the other hand the method of proof must depend upon the facts which can be recalled. Hence in this study these two elements are considered as a single step. Corresponding to these four steps are the four fundamental abilities with which this study is concerned; namely,

1. The ability to draw a figure for a theorem,
2. The ability to state the hypothesis and conclusion accurately in terms of the figure,
3. The ability to recall additional known facts concerning the figure, and

4. The ability to select the necessary facts and to arrange them so as to produce a proof.

The purpose of this investigation is threefold:

1. To determine the relation of each of these four abilities to the teachers' marks. This should, in turn, determine either the extent to which teachers value these abilities or the degree to which they are able to base their marks on the things which they do value.

2. To determine the extent to which these abilities are developed in our high schools.

3. To develop tests which may be used for the purpose of diagnosis; that is for the purpose of determining whether or not the weaknesses of a class are due to the lack of development of one or more of these abilities.

II. Brief Statement of Plan of Procedure

It seems reasonable to suppose that school grades are measures of the abilities essential to the particular kind of work accepted by teachers as indicating a successful mastery of their subjects. If the teacher holds the pupils for original demonstrations and if, as we believe, the four abilities enumerated above are essential to such work, then the teacher's grades will, to a certain extent, be measures of these abilities. Hence if we can measure each of these abilities separately, the results should bear a definite relation to the teacher's grades. For this purpose four tests (to be referred to as Test A, B, C and D[1]) have been arranged. Test E[1] has been used to supplement the other tests, especially Tests C and D. In the case of each test all elements except the one tested for have been eliminated so far as it was possible. Each test was given to at least one thousand pupils in various schools in this country, and the coefficients of correlation between the pupils' test scores and their school grades have been computed. These coefficients have been taken as indices of the extent to which these abilities influence the teacher's grades. As standards to be used for the purpose of comparison the median score has been computed for each test.

[1] The attention of teachers who gave Test D or Test E to their pupils is called to the fact that the letters designating these two tests have, for the sake of logical order, been interchanged. If any teacher gave Test D she will find the data recorded under Test E and vice-versa.

III. The Tests

Limitations as to the Subject Matter.—Because of their more extensive knowledge of the subject it would be desirable to include in such a study those pupils who have completed all of plane geometry. But with this advantage there would come certain disadvantages. First, if we should include these more advanced pupils it would be necessary to eliminate those who have completed only two books of geometry, unless a second set of questions should be used for this group, but this would unduly increase the work of the investigation. Second, if we should thus limit the investigation to pupils having recently completed all of plane geometry, we would have a very specially selected group, since extensive elimination has taken place before this, and our data would not be representative. Third, the group of pupils having completed the whole of plane geometry is comparatively small and the difficulty in securing the desired number would be increased. On the other hand, it seems reasonable to suppose that by the time a class has completed the first two books of geometry the abilities in question will be sufficiently developed to play an important part in the study of the subject and to bear a definite relation to the teacher's grades if these abilities are fundamental to the kind of work which she demands. Also if the tests are to be used for the purpose of diagnosis, it is necessary that they should be given before the study of geometry is completed. Hence the tests have been limited to the subject matter included in the first two books of geometry.

Aim in Selecting Questions and Difficulties Involved.—In arranging the tests our aim has been to satisfy the following conditions: The tests shall include only such exercises as require a knowledge of the first two books of geometry for their solution. The material shall be as varied and as inclusive as possible. The exercises shall vary in degree of difficulty, proceeding by more or less equal intervals from a comparatively easy exercise to one which offers considerable difficulty. Original exercises only shall be included, thus avoiding, as nearly as possible, propositions which have received special attention in class. Each test shall be of such length that it can be given in a recitation period without involving the speed factor.

Certain difficulties have been met in the attempt to attain

these ideals. First, the nature of the material has made it impossible to include a large number of exercises in the preliminary tests,[1] and our field of choice has therefore been limited. Tests in arithmetic, spelling, composition, and other subjects are of such a nature that it is possible to give a large number of exercises in a few minutes. It is then possible to make a more satisfactory selection than we have been able to make. Also the number of high school pupils of the desired grade is comparatively small, and the course in geometry is so extensive that teachers are seldom justified in giving more than one period to such an investigation, thus making it difficult to secure a large group for the preliminary tests or to repeat a test if the first data failed to be satisfactory.

The Preliminary Tests.—From various texts five sets of thirty exercises each were selected on the basis of their probable fitness for the particular test in question. These exercises were carefully solved and ten were selected from each. Each set of ten exercises was arranged in a manner similar to that described on pages 10–19 and directions for giving the tests were formulated. Each test was then given to about thirty pupils in a representative school in order to determine, as far as possible, whether any changes should be made in the arrangement of the tests, in the wording of the exercises, in the number of exercises in each test, or in the directions for giving the tests. As a result of this trial the number of exercises in each test was reduced and the directions for giving Test D were revised.

During the second half of the school year 1915–16 each revised test was given to about two hundred pupils in order to secure data for the final selection of exercises. To avoid specially selected groups, it was desirable to give these preliminary tests to as great a variety of pupils as possible. Hence they were given in thirteen schools in the states of New York, New Jersey, Pennsylvania, West Virginia, Indiana, Illinois and Iowa. To guard further against a single test being given to a large number of pupils working under the same special conditions, two or more tests were given in each of the larger schools. This gave for each test data gathered from several groups of pupils working under varied conditions.

[1] See Preliminary Tests below.

The wide distribution of schools made it impossible for the author personally to conduct the tests in each school, although he did so in those schools which were near at hand. However, the simplicity of the tests made it possible to prepare directions which were simple and definite, and the results indicate that there has been no material variation from these directions. Two sets of instructions were sent to each school. One gave directions for certain preliminaries and for the disposal of the papers after the test had been given. The other gave specific directions for conducting the test. These instructions were the same as those on pages 21–24 except that the examiner was directed to note carefully the time required by each pupil, and there was no provision for collecting the papers after thirty minutes of actual work.

Final Selection of Exercises.—After the papers were carefully graded the exercises for the final tests were selected. From the experience gained in giving the preliminary tests it was evident that not more than thirty minutes were available for actual work during a recitation period. The time spent by each pupil and the amount of work completed indicated that not more than the following number of exercises could be included in each test: Test A, five; Test B, four; Test C, four; Test D, three; and Test E, four. Some pupils will not require the full thirty minutes for these tests but, as we desired to eliminate the speed factor, the time allowed should be ample for all pupils. After thus selecting the exercises for Test A and giving them to about three hundred pupils, it appeared that they were too easy to be effective and a new set of exercises was selected in the same manner as described above. The data from these preliminary tests have been omitted since the results of the final tests[1] will serve as an effective check on the validity of the choice of questions.

Description of the Tests.—Each exercise thus finally selected for a given test was printed on a separate sheet of paper with space below for the pupil's answer. These sheets together with a cover-sheet were bound at the top so as to give freedom in folding them back while the pupil worked. The content of the

[1] Pages 49–59.

cover-sheet for each test was the same as the following for Test A except for the letter designating the test:

TEST A.

Are you a boy or a girl?.....

Your Name.....

Town..... School.....

Date.....

Your Teacher's Name.....

Test A.

The purpose of this test was to measure the pupil's ability to draw accurate figures for theorems.

The exercises follow in the order in which they were given:

I

Draw the figure for the following proposition:

If two radii of a circle are perpendicular, and a tangent to the circle cuts these radii produced at points A and B, the other tangents drawn from A and B are parallel.

II

Draw the figure for the following proposition:

If two lines which are on opposite sides of a third line meet at a point of the third line, making the non-adjacent angles equal, the lines form one and the same line.

III

Draw the figure for the following proposition:

The perpendicular drawn from the point of intersection of the medians of a triangle to a line without the triangle is equal to one third the sum of the perpendiculars from the vertices of the triangle to the line.

IV

Draw the figure for the following proposition:

The bisectors of the interior and the exterior vertical angles of

a triangle meet the circumscribed circumference in the mid-points of the arcs into which the base divides the circumference, and the line joining those points is the diameter which bisects the base.

V

Draw the figure for the following proposition:

The bisectors of the angles included between the opposite sides (produced) of an inscribed quadrilateral intersect at right angles.

Test B.

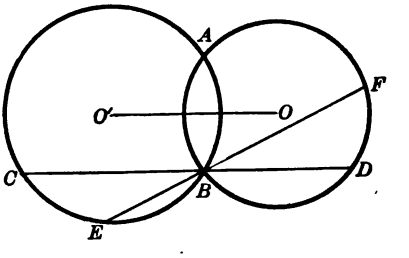
The purpose of this test was to determine the pupil's ability to state the hypothesis and conclusion of a theorem in terms of a given figure.

The exercises follow in the order in which they were given:

I

State what is given and what is to be proved in the following proposition:

If two circles intersect, the common secant drawn through one of the points of intersection and parallel to the line of centers is greater than any other common secant drawn through that point of intersection.



Given:[1]

To prove:

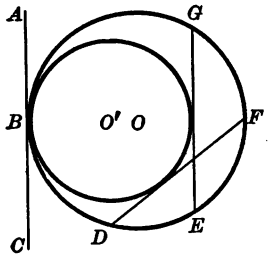
II

State what is given and what is to be proved in the following proposition:

If two circles are tangent internally and chords of the outer

[1] Ample space was left after "Given" and "To prove" for full answers.

circle are drawn tangent to the inner circle, that chord is greatest which is parallel to the common tangent.



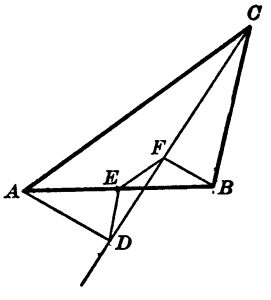
Given:

To prove:

III

State what is given and what is to be proved in the following proposition:

If from the extremities of a given side of a triangle perpendiculars are drawn to the bisector of the angle opposite that side, the lines connecting the feet of these perpendiculars to the midpoint of the given side are equal, and either is equal to half the difference of the other two sides of the triangle.



Given:

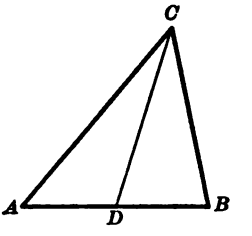
To prove:

IV

State what is given and what is to be proved in the following proposition:

An angle of a triangle is a right angle, an acute angle, or an obtuse angle, according as the median drawn from the vertex

of the angle is equal to, greater than, or less than one half of the opposite side.



Given:

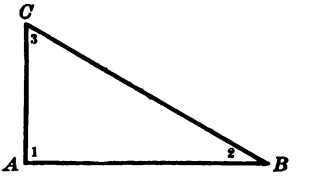
To prove:

Test C.

This test was arranged to measure the pupil's ability to recall known facts about figures when one or more facts are given

The questions follow in the order in which they were given:

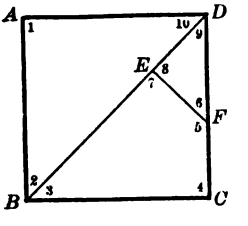
I



Given: Triangle ABC, $\angle 1$ = a right angle, and $\angle 3$ = two times $\angle 2$.

State as many more facts about the above figure as you can.

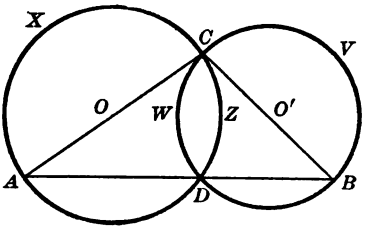
II



Given: The square $ABCD$, the diagonal BD, $EB = CD$ and EF is perpendicular to BD.

State as many more facts about the above figure as you can.

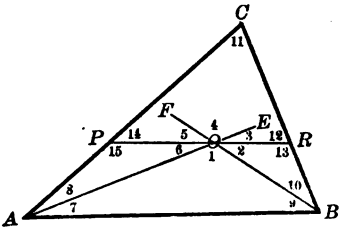
III



Given two circles O and O' intersecting in C and D, the diameters AC and CB, and the line AB.

State as many more facts about the above figure as you can.

IV



Given: Triangle ABC, AE bisects angle CAB, BF bisects angle ABC, AE and BF intersect in O, PR is drawn through O parallel to AB.

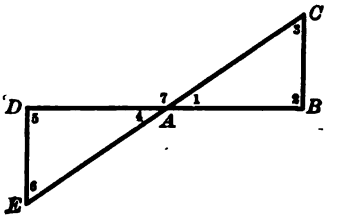
State as many more facts about the above figure as you can.

Test D.

The purpose of this test was to determine the pupil's ability to select and organize facts to produce a proof. At the top of each sheet there was a figure. Below was a statement of what was given and what was to be proved. To eliminate the factor tested for by Test C a list of "Other known facts" was given at the left hand side of the lower half of the sheet. To the right of this list was ample space for the pupil's proof. The list of "Other known facts" contained, among other facts, those essential to the proof. The pupil was free to select facts from this list if the figure did not suggest them.

The questions in the order in which they were given follow:

I



Given: $AB = AD$, ED is perpendicular to DB, CB is perpendicular to BD, and EC passes through A.

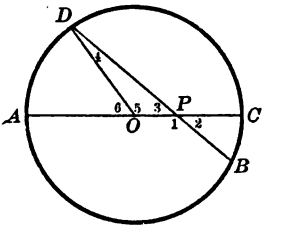
To prove that triangle ABC is congruent to triangle EAD.

Other known facts:

$\angle 6 = \angle 3$
$\angle 1 = \angle 4$
$\angle 4 + \angle 7 = 180°$
$\angle 2 = \angle 5$
ED is parallel to BC
$AC - CB < AB$

Proof:

II



Given: P is any point within the circle O, AC is a diameter through P, BD is any other chord through P, OD is a radius.

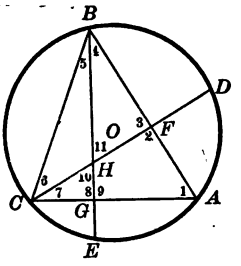
To prove that $AP > DP$.

Other known facts:

$\angle 6 = \angle 3 + \angle 4$
$PD - OD < OP$
$OD + OP > DP$
$\angle 2 = \angle 3$
$AO = OD$
$\angle 5 + \angle 6 = 180°$
$AO + OP = AP$

Proof:

III



Given: The triangle ABC inscribed in the circle whose center is O, CD is perpendicular to AB, BE is perpendicular to AC.

To prove that arc AE = arc AD.

Other known facts:	Proof:
$\angle 8 = 90°$	
$\angle 6$ is measured by half of arc BD	
$\angle 7$ is measured by half of arc AD	
$\angle 5$ is measured by half of arc EC	
$\angle 4$ is measured by half of arc AE	
$\angle 2 = 90°$	
$\angle 9 = \angle 5 + \angle 6 + \angle 7$	
$AC < AF + FC$	
$\angle 9 = 90°$	
$AC - AF < FC$	
$\angle 2 = \angle 4 + \angle 5 + \angle 6$	
$\angle 2 = \angle 8$	

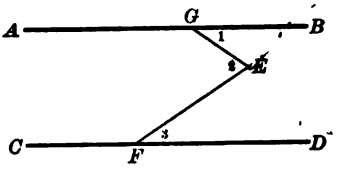
Test E.

The purpose of this test was to add further evidence to the results of Tests C and D by determining the pupil's ability to draw auxiliary lines. It is assumed that if a pupil can draw the auxiliary lines necessary for the proof of a theorem he must have in mind a definite proof and the facts necessary to that proof. The validity of this assumption will be discussed later.[1] However, if this is a true assumption a test requiring the pupils to draw the lines necessary to the proof of a theorem is a measure of his ability to recall facts, and select and arrange them to produce the proof.

The questions follow in the order in which they were given:

[1] Page 50.

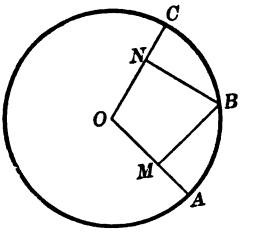
I



Given: AB is parallel to CD, and lines GE and FE meet in E.

Make any additional drawings that are necessary to prove that $\angle 2 = \angle 1 + \angle 3$.

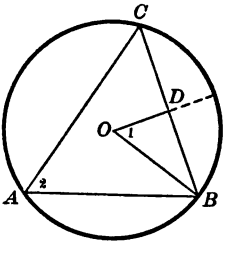
II



Given: The circle whose center is O, arc AB = arc BC, BM is perpendicular to AO, BN is perpendicular to OC.

Make any additional drawings that are necessary to prove that $BM = BN$.

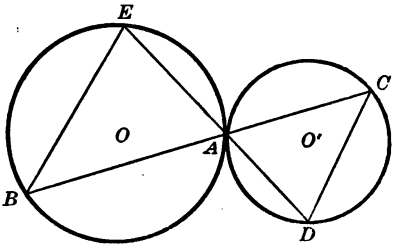
III



Given: Triangle ABC inscribed in a circle whose center is O, and OD perpendicular to CB.

Make any additional drawings that are necessary to prove that $\angle 1 = \angle 2$.

IV



Given: Circles O and O' tangent externally at A, BC and DE drawn through the point of tangency and terminating in the circumferences.

Make any additional drawings that are necessary to prove that BE is parallel to DC.

IV. Giving the Tests

Class of Pupils Tested.—For reasons stated on page 8 only those pupils who had recently completed the first two books of geometry were tested. Some teachers proposed giving the tests to small groups of their brighter pupils. In order to avoid such a specially selected group, all pupils of a school completing the required work at a given time took the same test. It was not possible to give all the tests in the same school for this would have required five days.

Since all types of pupils should be included, it was desirable to give the tests only to classes in which there had been no elimination. This was not always possible without seriously delaying the investigation. Some schools spend more than one semester on the first two books of geometry and in such schools some elimination had usually taken place before the required work had been completed. If these schools had been excluded, the number of available pupils would have been reduced and the difficulty in securing the desired number greatly increased. Hence such schools have been included. The probable effect of the resulting elimination will be discussed in connection with the data.[1]

Time when the Tests were Given.—The tests could not well be given at the close of the year 1915–16 for, as indicated above,

[1] Page 55.

in many schools the pupils who began the study of geometry at the middle of that year did not complete the required two books until the following year. Hence it was decided to give the tests in the fall of 1916 to pupils who completed the required work at that time or at the close of the preceding year; in January, February and March of 1917 to pupils who completed the work about that time; and again at the close of the school year 1916–17. In order to overcome the effect of the long summer vacation the classes which completed the work in the spring of 1916 did not take the tests until they had continued their study of geometry for about a month in the fall of 1916.

Schools in which the Tests were Given.—In order to guard further against a special selection of pupils each test was given in several schools varying as much as possible in their nature and location. In all, the final tests were given in sixty-three schools. The Roman numerals[1] from I to LXIII have been assigned to these schools, and throughout the discussion reference to a school will be by the number assigned to it. The geographical distribution of the schools follows:

New York: Schools I, II, III, VIII, IX, XVIII, XX, XXVI, XXVII, XXXI, XXXIII, XXXV, XLI, XLII, XLIV, XLV, XLVI, XLVII, XLVIII, XLIX, LIV, LXIII.

Rhode Island: School XXI.

New Jersey: Schools VII, XXXVII, LVII.

Pennsylvania: Schools VI, XII, XIII, XIV, XV, XIX, XXII, XXV, XXVIII, XXIX, XXXII, XXXIV, XL, XLIII, L, LI, LIII, LV, LIX, LX, LXI.

West Virginia: School XXXIX.

Ohio: School LVIII.

Indiana: Schools IV, X, XI, XVI, XVII, XXX, XXXVI, LII, LVI.

Illinois: Schools V, XXXVIII, LXII.

Iowa: School XXIII.

Minnesota: School XXIV.

The schools in which each test was given will be indicated in the tables containing the data.[2]

[1] The principal or the head of the department of mathematics may obtain the number of his school by addressing the author.

[2] Pages 46–49.

Means of Securing Uniformity in Giving the Tests.—In most investigations of this nature uniformity requires that the same person conduct the test in all schools. The wide distribution of the schools tested made this impossible. However, as indicated on page 10, it was possible to formulate simple and definite directions which insured a fair degree of uniformity. The author is personally acquainted with many of the teachers who gave the tests and knows that they can be relied upon to follow directions carefully. Letters of inquiry written by examiners before the tests were given and comments often returned with the papers indicated the examiner's desire to follow directions. In only two cases did the returns from any of the schools indicate a variation from the directions and the papers from these schools were rejected.

The directions for giving the tests were as follows:

Instructions to the Examiner.

1. Before going to class read the "Class Rules" carefully and then during the test follow them without any variation.

2. On the day preceding the test remind the pupils to bring pencils, rulers, and compasses to the test, unless such instruments are kept in the class room.

3. Do not give any one of the tests unless you have thirty minutes for actual work by the class.

4. Fill out one of the enclosed grade cards for each class tested. Give the names of the pupils in the class and the final school grade which each received in the first two books of geometry. If the final grades have not been made out retain the grade card until they have and then send the grades to me. Do not fail to send these grades as all other data will be useless without them.

5. When all the papers of a class have been returned to you place the grade card for that class on top of the papers and bind them together with the elastics. Do not roll the papers; keep them in a flat package.

6. Do not grade the papers. Look over them if you care to see what your pupils were able to do with the exercises but do not place any marks on them.

7. When you have finished giving the test in the school return the papers (Collect, by express) to J. H. Minnick, 811 N. 40th St., Philadelphia, Pa.

Class Rules for Test A

1. See that each pupil is supplied with pencil, ruler and compass.

2. *Read to the Pupils.*—I am going to give you some geometry exercises. In order that all of you may have the same chance I want you to start at the same time. Do not open the set of questions which you are about to receive until I give the signal to begin work by tapping on the desk.

3. Distribute the questions.

4. Have pupils fill out blanks on the cover sheet of the questions.

5. *Read to the Pupils.*—At the top of each sheet which you have received there is an exercise from geometry. When the signal is given to begin work fold back the cover sheet, read one of the exercises carefully, and then in the space below draw the figure for the exercise. Then read another exercise and draw the figure. Continue in this manner until you have drawn the figures for all the exercises. Draw the figures as accurately as possible but you need not make actual constructions. Do not attempt to prove the exercises. All I want to know is whether you can draw the figure. You may do the exercises in any order you care to. You will have thirty minutes in which to complete your work.

6. Give the pupils a chance to ask questions concerning the instructions but do not reveal the content of the questions by your answers.

7. Note the time and then give the signal, thus: "Ready," and then tap on the desk with your pencil.

8. In the case of any irregularity on the part of any pupil during the test make a note on the cover sheet of his questions indicating the exact nature of the irregularity.

9. Collect all papers promptly at the close of thirty minutes of actual work.

The "Class Rules" for the other tests were similar to those for Test A. The first item was varied according to the instruments needed, and the fifth item was varied as follows:

For Test B.—

5. *Read to the Pupils.*—At the top of each sheet which you have received there is an exercise from geometry. Just below this is

the figure for the exercise. When the signal to begin work is given fold back the cover sheet, read one of the exercises carefully, and then in the space below state what is given and what is to be proved in the exercise. Then proceed in the same manner with each of the other exercises. Be sure that you have stated fully every fact that is given. Do not attempt to prove the exercises. All I want to know is whether you can tell what is given and what is to be proved. You may do the exercises in any order you care to. You will have thirty minutes in which to complete your work.

For Test C.—

5. *Read to the Pupils.*—At the top of each sheet which you have received there is a geometrical figure. Below this there is a statement of what is given. When the signal to begin work is given fold back the cover-sheet, read carefully what is given in one of the exercises and then in the space below state as many more facts about the figure as you can. When you have done this proceed in the same manner with another exercise. You may do the exercises in any order you care to. You will have thirty minutes in which to complete your work.

For Test D.—

5. *Read to the Pupils.*—At the top of each sheet which you have received there is a geometrical figure. Below this figure there is a statement of what is given and what is to be proved. Below this and on the left hand side of the sheet there is a number of other facts about the figure some of which may guide you in proving the exercise. When the signal to begin work is given fold back the cover-sheet, read carefully what is given and what is to be proved in one of the exercises, and then in the space at the bottom and to the right of the sheet give a complete proof of the exercise, but you need not give reasons or authorities for the different steps in your proof. Refer to the facts stated on the left hand side of the sheet as much as you care to and use any of them that will help in your proof. When you have completed the proof of this exercise proceed in a similar way to prove the other exercises. You may do the exercises in any order you care to. You will have thirty minutes in which to do your work.

For Test E.—

5. ***Read to the Pupils.***—At the top of each sheet which you have received there is a geometrical figure. Below this figure there is a statement of what is given and what is to be done. When the signal to begin work is given fold back the cover-sheet, read carefully what is given and what is to be proved in one of the exercises, and then make any additional drawings that are necessary to prove the exercise. Thus, if in the triangle ABC, $AC = BC$ (place drawing on the board) and we are to prove that $\angle A = \angle B$ we may draw CD to the mid-point of AB. When you have completed this exercise proceed in a similar way to do the other exercises. Do not write any explanations on your papers and do not prove the exercises. All I want to know is whether you can make the correct drawings. You may do the exercises in any order you care to. You will have thirty minutes in which to do your work.

In an investigation including so many widely distributed schools it is impossible to secure complete uniformity of physical conditions. However, teachers were asked to report any conditions which would influence the results of the tests, and from these reports it is believed that there were no disturbing elements of significance.

V. Scoring the Papers

General Statement of Method.—Two scores were kept for each exercise. One was the per cent. of necessary elements given correctly by the pupil; the other was the number of incorrect and unnecessary elements given. The former will be referred to as the *positive* score and the latter as the *negative* score. We know of no valid method of combining these. Teachers do combine them in some way or other. For example, if in stating the hypothesis of a theorem a pupil gives every fact correctly but also includes one statement which is not given, some teachers offset the error by one of the correct statements and give the same grade as if the pupil had made no incorrect statements but had omitted one of the necessary facts. We do not know that any two such statements are together equal to no statement at all, and any such combination of correct and incorrect statements is arbitrary. It has been suggested by some teachers that the best way to meet this situation is to disregard the negative scores altogether. Such a procedure assumes one of two things; namely, either that the positive

scores alone give the same distribution of pupils as the positive and negative scores combined, or that the misconceptions upon which the negative scores are based have no significance so far as this study is concerned. We shall now show that neither of these assumptions is true and that we therefore can not neglect the negative scores.

For this purpose we have selected the data from schools LII, LIV and LVI in each of which Test A was given. These schools present three rather typical forms of distribution and are a fair representation of conditions resulting not only from Test A but from each of the other tests. The 48 pupils of school LII were arranged according to their positive scores, and then, as nearly as possible, divided into five equal groups. As 48 is not exactly divisible by 5 some of the groups are necessarily larger than others. The extreme groups were made the smaller and the entire arrangement was made symmetrical with respect to the central group. Beginning with the poorest, these groups were numbered from one to five. In a similar way, the pupils were divided into groups according to their negative scores.

TABLE I.—*Relation between the distributions of the pupils of school LII according to their positive and their negative scores for Test A.*

		Negative.					
	A	5	4	3	2	1	D
Positive	5	3	5		1		9
	4	3	3	1	1	2	10
	3	2		5	2	1	10
	2	1	2	1	3	3	10
	1			3	3	3	9
	B	9	10	10	10	9	C

Table I shows the relation of the distributions given by each of the two groupings. The columns of squares represent the division of the class into fifths according to negative scores and the rows represent a like division according to positive scores.

Thus the first row means that the nine pupils constituting the highest one fifth of the class according to the positive scores are distributed according to the negative scores as follows: three are in the highest one fifth, five are in the next highest group, and one is in the group next to the lowest. In a similar way Tables II and III give the distributions of the pupils of schools LIV and LVI respectively.

TABLE II.—*Relation between the distributions of pupils of School LIV according to their positive and their negative scores for Test A.*

		Negative.					
A	5	4	3	2	1	D
Positive 5		2		1	1	4
4			1	1	2	4
3	1	2	2		1	6
2	1		1	2		4
1	2		2			4
B	4	4	6	4	4	C

TABLE III.—*Relation between the distributions of pupils of School LVI according to their positive and their negative scores for Test A.*

		Negative.					
A	5	4	3	2	1	D
Positive 5	2	4	2	3	1	12
4	1	4	2	2	4	13
3	2	1	4	4	2	13
2	5	1	3	2	2	13
1	2	3	2	2	3	12
B	12	13	13	13	12	C

Clearly a perfect positive correlation would result in the arrangement of all pupils along the diagonal AC, and a perfect negative correlation would give an arrangement of all pupils along the diagonal BD. In Table I there is a decided tendency to arrange the pupils along the diagonal AC, and therefore there is a fairly close correspondence between the distributions given by each set of scores. On the other hand, Table II shows that in School LIV there is actually a negative correlation between the two sets of scores. Again, Table III shows that in School LVI there is but a slight tendency to arrange the pupils along either diagonal. That is, there is little or no relation between the positive and negative grades in this school. Hence we cannot assume that the positive and negative scores give the same distribution of pupils. Furthermore when we attempt to determine each individual's rank we shall not be concerned with group tendencies but with individual variations, and even in those schools where there is a fairly close correspondence between the positive and negative scores there is considerable individual variation. Hence in no case can we neglect the negative scores; provided, of course, the misconceptions upon which they are based bear a vital relation to our study.

One purpose of this study, as we have previously stated, is to determine to what extent the abilities in question influence teachers' marks. These marks should be a measure of the pupil's *total* ability to do geometry and any element influencing this *total* ability should influence the teachers' marks. But unnecessary statements tend to confuse the pupil and an incorrect statement can lead only to an incorrect conclusion. Hence these misconceptions do bear a vital relation to our first purpose. Our third purpose is to furnish a means of educational diagnosis. If then, as we have just seen, these misconceptions interfere with the pupil's progress our investigation must concern itself with the negative scores. Hence, since we can not neglect the negative scores and since we know of no legitimate method of combining them with the positive scores, it seemed best to record them separately. This procedure is further justified by the fact that, for the purpose of diagnosis, it is essential that our results be as analytic as possible and that standards be established for each element entering into the study of geometry rather than for groups of elements.

The exercises of each test were weighted according to the average positive scores for all pupils taking the test.[1] Each pupil's paper was then given a positive grade based upon the values thus assigned to the various exercises. For reasons to be explained later the negative scores were not weighted.[2] Also as there is no upper limit to the number of incorrect and unnecessary elements that can be given, it is impossible to express the negative scores in per cents. Hence we have used as the pupil's negative score the total number of incorrect and unnecessary elements given in the entire test.

Means of Securing Uniformity of Scoring.—All papers were scored by the author. In each test the first exercise was scored for the entire school, then the second, and so on, until all the exercises had been marked for the given school. Before beginning to mark the papers, each exercise was carefully solved and the number of necessary steps noted. This number was used as a basis for expressing the pupils' positive scores for that exercise in per cents. Some of the exercises admit of more than one solution. In such cases a copy of each new solution found among the papers was kept for reference, and a pupil's positive score was based on the number of necessary steps in the method which he had chosen. If any answer gave peculiar difficulty a memorandum of how it was scored was kept as a guide in all similar cases.

There will, no doubt, be a difference of opinion as to the number of steps necessary for a complete solution of some of the exercises, but the details given in the following pages are the results of experience gained in scoring about one thousand papers from the preliminary tests, and any one caring to compare his results with those of this investigation should carefully follow this method of scoring.[3]

Scoring Each Test.—We shall now describe in detail the scoring of each test.

Test A.—The figure necessary for the correct solution of each exercise of Test A is given below. With each figure is a statement of the points necessary for a complete drawing.[4]

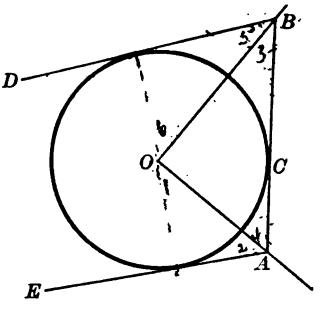
[1] Pages 43–49.

[2] Page 44.

[3] For a condensed statement of directions for scoring papers see page 97.

[4] For the exercises see page 11.

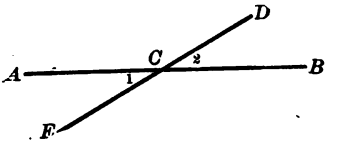
I



Circle O 1
OA a radius 1
OB { a radius perpendicular to OA } 2
AB { tangent to O cutting OB at B cutting OA at A } 3
BD { tangent to O through B } 2
AE { tangent to O through A } 2

Total number of points 11

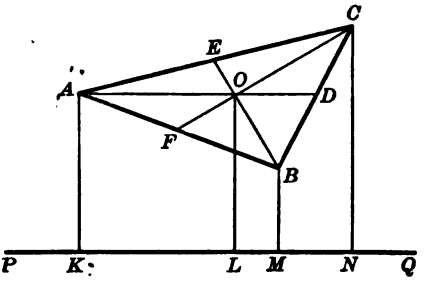
II



AB a straight line 1
CD a straight line 1
EC { a straight line opposite AB from CD meeting CD on AB } 3
$\angle 1 = \angle 2$ 1

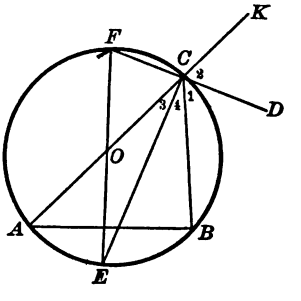
Total number of points 6

III



Triangle ABC	1
AD { through A / to mid-point of BC }	2
BE similar to AD	2
CF similar to AD	2
PQ { a straight line / outside of ABC }	2
AK { from A / perpendicular to PQ }	2
BM similar to AK	2
CN similar to AK	2
OL similar to AK	2
Total number of points	17

IV



Circle O	1
ABC { triangle / inscribed }	2
AC produced to K	1

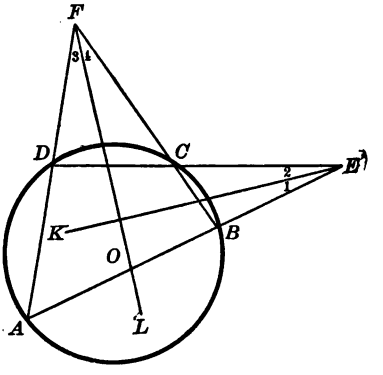
CD { through C / making $\angle 1 = \angle 2$ } 2

CE { through C / making $\angle 3 = \angle 4$ } 2

EF { through E / through F } 2

Total number of points 10

V



Circle O 1

$ABCD$ { quadrilateral / inscribed } 2

AD and BC produced to meet 1

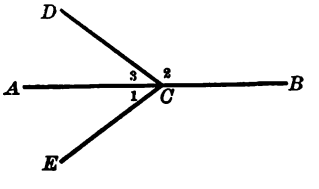
DC and AB produced to meet 1

EK { through E / making $\angle 1 = \angle 2$ } 2

LF similar to EK 2

Total number of points 9

As an illustration of this method of scoring, suppose a pupil drew the following figure for exercise II.



The correct points in this drawing are as follows:

AB a straight line	1
CD a straight line	1
EC { a straight line opposite AB from DC meeting DC on AB }	3
Total number of correct points	5

Incorrect drawing:

$\angle 1 = \angle 3$	1

The total number of correct points should be 6. The pupil has five of these correct and he has one error. Hence his positive score is 83, and his negative score is 1.

Certain peculiarities should be noted. If the pupil omitted the letters A and B from his figure for exercise I nothing was deducted from his positive score. If, however, these letters were incorrectly used they were counted in determining the negative score. If in the figure for exercise III the medians were not produced to the mid-points of the sides of the triangle but would pass through these points if produced, they were counted as correct. Also if the figure was drawn so that two of the perpendiculars to the line PQ coincided full credit was given for these coincident perpendiculars. In exercise IV some pupils drew the bisectors of the exterior and interior angles at each of the three vertices. In such cases the drawings at two of the vertices were counted as unnecessary. The pupils sometimes produced the opposite sides of the inscribed quadrilateral of exercise V in the direction in which they would not meet. This was counted as an unnecessary drawing.

Test B.—The answer to each exercise of Test B and the number of necessary statements in each answer are given below.[1]

I

Given:

Circles O and O'	2
B a point of intersection of O and O'	1
OO' the line of centers	1

[1] See pages 12–14 for exercises and figures.

CB { a common secant / through B / parallel to OO' } 3

EF { a common secant / through B / not parallel to OO' } 3

Total number of points 10

To prove:

$CB > EF$ 1

II

Given:

Circles O and O' internally tangent 3

BC the common tangent 1

EG { chord of circle O / tangent to circle O' / parallel to AC } 3

DF { chord of circle O / tangent to circle O' / not parallel to CB } 3

Total number of points 10

To prove:

$EG > DF$ 1

III

Given:

Triangle ABC 1

Angle BCA 1

CD bisects angle BCA 1

AB the side opposite angle BCA 1

BE perpendicular to CD 1

AD perpendicular to CD 1

F the mid-point of AB 1

Lines FE and FD 2

Total number of points 9

To prove:

$FE = FD$ 1

FE or $FD = \frac{1}{2}(AC - CB)$ 1

Total number of points 2

IV

Given:

Triangle ABC	1
Angle ABC	1
Median CD	1
AB the side opposite angle ACB	1
$CD = \frac{1}{2}AB$,	1
$CD > \frac{1}{2}AB$, or	1
$CD < \frac{1}{2}AB$	1
Total number of points	7

To prove:

Angle ACB is a right angle,	1
Angle ACB is an acute angle, or	1
Angle ACB is an obtuse angle	1
Total number of points	3

The unweighted positive score for each exercise was obtained by grading the hypothesis and conclusion each on a scale of 100 and then taking the average of the two grades. A statement was considered as correct only when it was given in terms of the figure. Such general statements as, "Given two intersecting circles" were counted in determining neither the positive nor the negative score. Pupils frequently include a part of the hypothesis in the form of a modifying phrase or clause in the statement of the conclusion. Generally this should not be permitted as it causes the pupil to lose sight of the parts of the hypothesis given in the conclusion. For this reason any part of the hypothesis given in the conclusion was not counted as correct,[1] nor was it included in the count for the negative score. An illustration will make the method of scoring clear. Suppose a pupil answered exercise III as follows:[2]

Given: The triangle ABC with the bisector of angle ACB, BF and AD are perpendicular to CD.

To prove: The lines FE and ED connecting the mid-point of AB with the feet of the perpendiculars BF and AD are equal and either equals $\frac{1}{2}(AC - AB)$:

Points correctly and specifically stated in hypothesis:

[1] An exception was made to this rule in exercise IV. See pages 35–36.

[2] Page 13.

ABC is a triangle 1
Angle ACB 1
BF is perpendicular to CD 1
AD is perpendicular to CD 1

Number of correct points in hypothesis 4
Per cent. of points correct in the hypothesis 44

Points correctly and specifically stated in the conclusion.

FE equals ED 1
Per cent. of points correct in conclusion 50
Average score for hypothesis and conclusion 47

Incorrect statements in the answer:

FE equals $\frac{1}{2}(AC - AB)$ 1
ED equals $\frac{1}{2}(AC - AB)$ 1

Total number of incorrect points 2

Hence the positive score is 47 per cent. and the negative score is 2. The statement concerning the bisector of angle ACB is not specific and is therefore not counted. The statements concerning the lines FE and ED and the mid-point of AB are not counted because they are involved in the statement of the conclusion.

Certain special cases should be noted. Care was taken not to count the same lack of specific statement twice. For example, if in exercise IV a pupil made the following statement, "Given the bisector of angle ACB, and AD perpendicular to the bisector of ACB," there are seemingly two points in which he failed to be specific. He did not name the bisector of angle ACB nor did he name the line to which AD is perpendicular. If, however, he had named the bisector of angle ACB the second statement would have been specific. Hence he should receive credit for the second statement. Freedom of expression was permitted as long as the pupil made a specific statement of each point in the hypothesis and conclusion. Thus in exercise II he was not required to say that DF is not parallel to CB. Any statement clearly distinguishing DF from GE was accepted. In exercise IV there are three conclusions each dependent upon a separate part of the hypothesis. Pupils experience considerable difficulty in getting a clear statement of this case if they are required to separate completely the hypothesis and conclusion. Hence for a statement such as the following:

To prove that

1 Angle ACB is a right angle if $CD = \frac{1}{2}AB$,
2 Angle ACB is an acute angle if $CD > \frac{1}{2}AB$,
3 Angle ACB is an obtuse angle if $CD < \frac{1}{2}AB$,

credit was given for a perfect statement of the conclusion and for three points in the hypothesis. It may seem that the conclusion of exercise III should count as three points; namely, $FE = FD$, $FE = \frac{1}{2}(AC - CB)$, and $FD = \frac{1}{2}(AC - CB)$. But the first and either of the other two statements are equivalent to the remaining statement and since the pupils seemed to have this clearly in mind the conclusion was counted as two points.

Test C.—In Test C[1] the pupil was free to give any facts which he could recall concerning the given figure. The pupils' statements varied so greatly that it is impossible to give model answers for the various exercises of this test. We do not know how many facts about any one of these figures a pupil should be able to give. Therefore, we have no exact basis for computing the positive scores in per cents and it is necessary to select one arbitrarily. The smallest number of facts stated correctly by any one of the highest ten per cent. of all pupils taking the test does not seem too high a standard to set for a perfect answer. However, it is seldom possible to make this exact division of the pupils. For example, suppose the following condition to exist: If we take the highest group of pupils such that the smallest number of facts given by any one of them is nine we include less than ten per cent. of all the pupils; but if we increase this group until the smallest number given by any one is eight we include more than ten per cent. It is then impossible to select the number of facts given by exactly the highest ten per cent. In all such cases the larger number of facts was selected. That is, the basis for computing the positive score in per cents was the smallest number of facts given by any one of the highest group of pupils, this group not to exceed ten per cent. of all pupils taking the test but to be as nearly ten per cent. as possible. The number of correct facts given by each pupil has been carefully noted and the data is given in Table IV. In order to indicate how nearly a constant condition has been obtained this table has been arranged in a cumulative way. Thus, the first

[1] Pages 14–15.

line gives the data for school VII, the second line for schools VII and VIII, etc., the last line giving the combined data for all the schools in which the test was given. In column a under I is the least number of facts for exercise I given by any one of the highest group of pupils which does not exceed ten per cent. of all the pupils taking the test but is as nearly ten per cent. as possible. In column b under I is the per cent. of all the pupils who gave that number of facts correctly. Thus, the seventh line indicates that, for exercise I, eight or more facts were given correctly by each pupil of the highest 9.8 per cent. of those taking the test, and 8 is the smallest number of facts that can be taken without including more than ten per cent. of the pupils. An examination shows that enough pupils have been tested to give fairly constant results for all the exercises with the possible exception of exercise II. Therefore the numbers of correct facts accepted as perfect positive scores for the exercises of Test C were 8, 30, 7 and 18 respectively.

TABLE IV.—*Least number of correct facts given for each exercise of Test C by any one of the highest group which does not exceed*[1] *ten per cent. of all pupils taking the test but is as nearly ten per cent. as possible.*

Exercise	I		II		III		IV		Number of Pupils
School	a	b	a	b	a	b	a	b	
VII	11	10.0	32	6.7	9	7.7	21	10.0	30
VIII	9	6.6	34	9.1	8	8.0	19	9.0	578
IX	9	6.0	33	9.4	8	7.5	19	8.4	651
X	9	5.9	33	8.9	8	7.3	19	8.2	682
XI	9	5.3	31	10.0+	7	10.0+	19	8.2	710
XII	8	10.0	31	10.0	7	9.3	18	10.0	795
XIII	8	9.8	31	9.6	7	9.0	18	10.0+	844
XXVIII	8	9.6	31	9.2	7	8.7	18	9.8	882
XXXII	8	9.5	31	9.0	7	8.7	18	9.2	908
XXXIV	8	10.0+	30	9.7	7	9.4	18	9.4	993
XXXVII	8	9.8	30	9.6	7	9.4	18	10.0	1019
XXXVIII	8	9.8	30	9.5	7	9.4	18	10.0	1047

In Test C a statement was counted as correct only when it gave accurately and concretely some relation between parts of the figure or when it gave the value of some magnitude correctly. The mere naming of the parts of a figure (*e. g.*, AB is a chord) was counted in determining neither the positive nor the negative grade. General statements, such as "The sum of two sides of a

[1] In this table 10.0+ indicates that slightly more than ten per cent. of the pupils gave the corresponding number of correct facts.

triangle is greater than the third side," are useless in the demonstration of a proposition unless the pupil can show how they apply to a given figure. Hence only facts stated in terms of the figure were counted as correct. However, statements given in general terms were counted in determining the negative grade only when they were incorrectly given. Pupils sometimes made additional drawings and then gave facts concerning the new figure. Such facts were eliminated for two reasons. Many pupils who could have given such facts did not because the test did not call for them. Hence their results would not have been comparable with the results of those who did give such statements. Second, there is no limit to the number of such facts since there is no end to the drawings which could be added. If a pupil made a continued statement such as $a = b = c = d$ or $x > y > z$ credit was given for the full number of facts involved. On the other hand care was taken not to give credit twice for a fact which was repeated in the same or slightly different form. Thus $a + b = c$ and $a = c - b$ express the same relation. Likewise the statements $a > b$ or c, and $c < a$ or b repeat the relation between a and c.

Test D.—In this test the pupil was asked to produce the proof for the exercises. As each exercise admitted of two or more proofs and the pupil was free to select any proof he desired, it is necessary to consider the different proofs possible for each exercise. The various proofs found in the papers follow[1]:

I

(*a*)	$AB = AD$	1
	$\angle 1 = \angle 4$	1
	$\angle 2 = \angle 5$	1
	$\triangle ADE = \triangle ABC$	1
	Number of necessary steps	4
(*b*)	ED is parallel to BC	1
	$\angle 2 = \angle 5$	1
	$\angle 1 = \angle 4$	1
	$AB = AD$	1
	$\triangle ADE = \triangle ABC$	1
	Number of necessary steps	5

[1] For the exercises see pages 15–17.

(*c*) Make DA coincide with AB 1
$\angle 1 = \angle 4$ 1
AE takes the direction of AC 1
$\angle 5 = \angle 2$ 1
DE takes the direction of BC 1
Point E falls on point C 1
$\triangle ADE = \triangle ABC$ 1

Number of necessary steps 7

II

(*a*) $DO + OP > DP$ 1
$DO = AO$ 1
$AO + OP > DP$ 1
$AO + OP = AP$[1] 1
$AP > DP$ 1

Number of necessary steps 5

(*b*) Draw AD 1
$AO = DO$ 1
$\angle 9 = \angle 8$ 1
$\angle 9 + \angle 4 > \angle 8$ 1
$AP > DP$ 1

Number of necessary steps 5

III

(*a*) $\angle 2 = \angle 9$ 1
$\angle 1 = \angle 1$ 1
$\angle 4 = \angle 7$ 1
$\angle 4$ is measured by $\frac{1}{2}AE$ 1
$\angle 7$ is measured by $\frac{1}{2}AD$ 1
Arc AE = arc AD 1

Number of necessary steps 6

[1] Sometimes when this statement was omitted it was clear that the pupil had it in mind. In such cases credit was given for the step.

(*b*) $\angle 8 = \angle 3$ 1
$\angle 10 = \angle 11$ 1
$\angle 7 = \angle 4$ 1
$\angle 7$ is measured by $\frac{1}{2}AD$ 1
$\angle 4$ is measured by $\frac{1}{2}AE$ 1
Arc AE = arc AD 1

Number of necessary steps 6

(*c*) $\angle 1 + \angle 2 + \angle 7 = 180°$ 1
$\angle 1 + \angle 4 + \angle 9 = 180°$ 1
$\angle 1 + \angle 2 + \angle 7 = \angle 1 + \angle 4 + \angle 9$ 1
$\angle 1 = \angle 1$[1] 1
$\angle 2 = \angle 9$ 1
$\angle 7 = \angle 4$ 1
$\angle 7$ is measured by $\frac{1}{2}AD$ 1
$\angle 4$ is measured by $\frac{1}{2}AE$ 1
Arc AE = arc AD 1

Number of necessary steps 9

(*d*) $\angle 2 = \angle 4 + \angle 5 + \angle 6$ 1
$\angle 9 = \angle 5 + \angle 6 + \angle 7$ 1
$\angle 2 = \angle 9$ 1
$\angle 4 + \angle 5 + \angle 6 = \angle 5 + \angle 6 + \angle 7$ 1
$\angle 5 + \angle 6 = \angle 5 + \angle 6$[1] 1
$\angle 4 = \angle 7$ 1
$\angle 4$ is measured by $\frac{1}{2}AE$ 1
$\angle 7$ is measured by $\frac{1}{2}AD$ 1
Arc AE = arc AD 1

Number of necessary steps 9

(*e*) CD is perpendicular to AB 1
BE is perpendicular to AC 1
$\angle 4 = \angle 7$ 1
$\angle 4$ is measured by $\frac{1}{2}AE$ 1
$\angle 7$ is measured by $\frac{1}{2}AD$ 1
Arc AE = arc AD 1

Number of necessary steps 6

[1] If this statement was omitted but clearly in the mind of the pupil credit was given for it.

(*f*) $\angle 3 = 90°$ 1
$\angle 4 + \angle 11 = 90°$ 1
$\angle 8 = 90°$ 1
$\angle 7 + \angle 10 = 90°$ 1
$\angle 4 + \angle 11 = \angle 10 + \angle 7$ 1
$\angle 11 = \angle 10$ 1
$\angle 4 = \angle 7$ 1
$\angle 4$ is measured by $\frac{1}{2}AE$ 1
$\angle 7$ is measured by $\frac{1}{2}AD$ 1
Arc AE = arc AD 1

Number of necessary steps 10

(*g*) $\angle 8 = \angle 3$ 1
$\angle 8$ is measured by $\frac{1}{2}(BC + AE)$ 1
$\angle 3$ is measured by $\frac{1}{2}(BC + AD)$ 1
$BC + AE = BC + AD$ 1
$AE = AD$ 1

Number of necessary steps 5

(*h*) $\angle 7$ is a complement of $\angle 10$ 1
$\angle 4$ is a complement of $\angle 11$ 1
$\angle 10 = \angle 11$ 1
$\angle 4 = \angle 7$ 1
$\angle 4$ is measured by $\frac{1}{2}AE$ 1
$\angle 7$ is measured by $\frac{1}{2}AD$ 1
$AE = AD$ 1

Number of necessary steps 7

The number of necessary steps in the proof of an exercise was used as the basis for computing the positive score. The pupil was free to select any method of proof he desired and the number of necessary steps varied with his choice. In each case the number of necessary steps in the proof chosen was taken as the basis. The numbers used for each method found in the papers are given in connection with the proofs on pages 38–41. These numbers may be slightly varied depending upon the number of statements which are implied but not expressed. The selection of the above numbers was based on experience gained in grading a large number of papers, and further experience seems to justify

this selection. If a proof was incomplete the pupil was given credit for the number of correct facts given. If he had not carried the proof far enough to show what method he had in mind the scorer completed the proof with the least possible number of steps and used that number as a base for computing the positive score. If a statement was given out of its logical order it was counted incorrect. If, however, a conclusion was followed by the facts leading up to it and the relation was indicated by "since," "for," or some other like expression, full credit was given. For example, the following statements would be counted correct,

$$\triangle ABC = \triangle DEF,$$

since

$$AB = DE,$$

$$BC = EF,$$

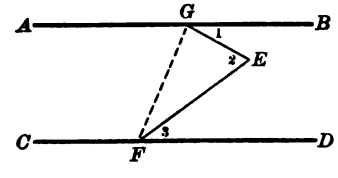
and

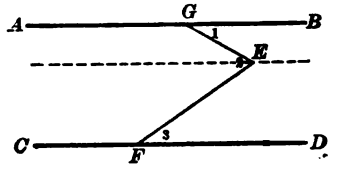
$$\angle B = \angle E.$$

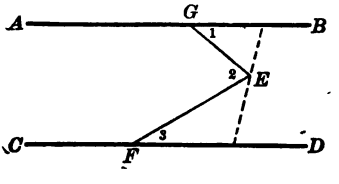
The pupil was asked to omit the authorities and reasons for the steps in his proof. If such authorities were given they were not used in determining either the positive or the negative score. The pupil often made a correct statement and later repeated it, apparently to call attention to it. Such repetitions were disregarded.

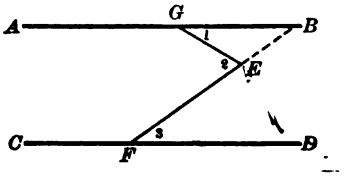
Test E.—In this test[1] the positive score was either one hundred or zero. If a drawing made a proof possible it was one hundred. For any other drawing it was zero. The following drawings making a proof possible were given by the pupils.

I

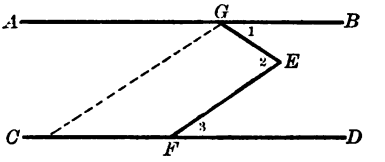




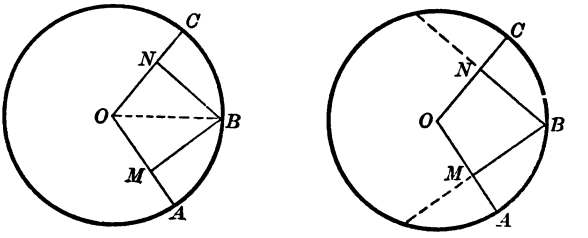




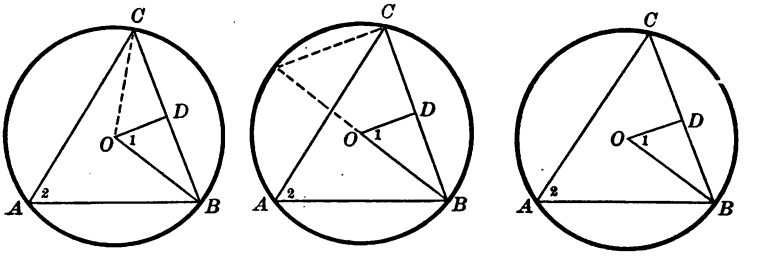
[1] For the exercises see pages 17–19.



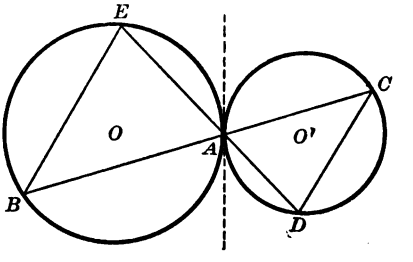
II



III



IV



Weighting the Exercises.—As the exercises of any one of the tests differ in degree of difficulty with respect to both the positive and negative scores, different values should be assigned to each exercise. Thus, a pupil should receive more credit for a correct drawing for exercise V of Test A than for a correct drawing

for exercise I of the same test. Certain difficulties present themselves which we must now consider.

As the positive and negative scores for each exercise cannot be combined, a single weight cannot be assigned for the two elements. The incorrect and unnecessary steps introduced into the solution of a given exercise varied greatly. One error occurred more frequently than another, but the same error seldom occurred in a large number of papers. Therefore we do not have enough material to weight each error separately. Moreover such a procedure would involve an undue amount of labor as there was no limit to the number of different errors which could be introduced. Also, we have no means (such as the per cents of total number of possible errors) of comparing the negative values of the exercises of a test.[1] Therefore the exercises have not been weighted according to the negative scores. The total number of errors made by a pupil in answering all the exercises of a test was taken as the final negative score.

The correct solution of any of the exercises, excepting those of Test E, involves several steps which differ in degree of difficulty. If we try to weight these steps separately on the basis of the per cent. of pupils giving them correctly, complications arise from the facts that in Tests C and D the statements required for a correct solution were not always the same for a given exercise. Further in Test D the steps are so related that it is impossible to say that a difficulty lies wholly within any one of them. Hence this basis for weighting seems impracticable. There is also a possibility of weighting each statement according to its relative geometrical value but we do not know how to determine this value. Hence it was decided to weight each exercise as a whole according to the positive scores.

The question of securing data under the same conditions for each of a set of questions also involved a difficulty. An exercise occurring in a series of exercises has two types of difficulty. One is its *intrinsic* difficulty due to its own peculiarities; the other may be called its *place* difficulty due to its position in the series. Fatigue, suggestion from a preceding exercise, distraction caused by a preceding difficulty, and encouragement due to preceding success are some of the factors which influence this latter type of difficulty. In order to eliminate this *place* difficulty

[1] Page 28.

investigators sometimes give a series of questions in one order and then reverse the order and average the grades of each question for the two trials. This procedure may eliminate one or more of the influences due to position but the reversed order brings the pupils to a given question through a new succession of questions which introduces new elements. Hence we do not know that this method is equivalent to giving each exercise under the same conditions. For our purpose it is not even desirable that the *place* difficulty be eliminated. The exercises must be given in some order. This order will present its own peculiarities and the value assigned to each exercise should be dependent upon both its *intrinsic* and *place* difficulties. Hence the positive values assigned to the exercises of each test are based upon the results obtained by giving the tests in their final order without any reversal of that order.

We do not have sufficient data to locate a zero-point in the way it has been located in certain scales.[1] Due to extensive elimination during the earlier school years, the pupils tested constituted a specially selected group. Hence, the probability that any high school pupil will have zero ability in any phase of the work with which this study is concerned is very slight. Therefore, since no considerable number of pupils made a zero score in a given test, it is impossible to determine the exact point at which total inability to do any part of that test begins.

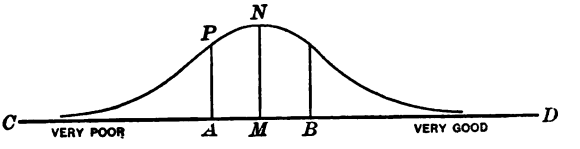


FIG. 1. Normal surface of frequency.

However, we may safely assume that distribution based on any of the abilities in question follows the same law as a distribution based on any other human trait. That is, if there were no eliminations distributions according to any one of these abilities would result in the normal frequency curve as shown in Fig. 1. The curve, when extended indefinitely in either direction from the median *MN*, continues to approach the line *CD*. The direction from left to right will be considered positive, and from right to

[1] Trabue, "Completion—Test Language Scales," p. 52.

left negative. If AP is drawn so that $AMNP$ is one fourth of the entire surface under the curve; that is, so that twenty-five per cent. of the cases fall between M and A, then AM is known as the P.E.[1] For practical purposes the curve meets the line CD at about 4.6 P.E. above and below M. If a question can be answered by all pupils above $-$ 4.6 P.E. the ability required is very slight. Therefore, the point $-$ 4.6 P.E. has been arbitrarily assumed as the zero-point. The weighted values assigned to the exercises of each test are proportional to the distances in P.E. above this arbitrary zero-point and they are such that their sum is one hundred. We shall now consider the weighting of the exercises of each test.

TABLE V.—*Average positive scores of each exercise of Test A.*

Schools	Exercises					Number of Pupils
	I	II	III	IV	V	
XXIII	77	76	65	52	30	88
XXV	79	75	65	50	30	158
XXXIII	80	76	62	49	29	173
XXXV	81	77	64	50	29	196
XXXVI	82	77	64	48	29	222
XXXIX	81	75	64	49	29	239
XL	82	75	65	51	30	286
XLI	84	74	68	50	31	334
XLII	86	76	79	50	32	385
XLIII	86	77	72	52	34	438
XLIV	87	78	73	55	36	490
XLV	88	79	75	55	37	541
L	88	79	75	55	37	555
LI	88	79	74	55	36	569
LII	88	79	73	55	37	617
LIII	87	79	73	55	36	688
LIV	87	80	73	55	36	710
LVI	87	79	72	54	35	773
LIX	87	79	72	54	36	802
LX	87	79	72	54	35	831
LXI	87	79	72	54	35	856
LXIII	88	79	71	54	34	944

Test A was given to 1,094 pupils but as school LXII gave the test to pupils who had completed all of plane geometry the data from this school were not used in weighting the exercises. Table V gives the average positive scores for each exercise of the test. If the values assigned to the exercises are to be reliable the number of pupils tested must be sufficient to eliminate chance

[1] Trabue, "Completion-Test Language Scales," pp. 30–35.

variations. In order to indicate how nearly this condition has been realized this table is arranged in a cumulative way. A study of the table shows that a fairly constant condition has been obtained, and perhaps the addition of more schools would not change the results materially.

TABLE VI.—*Positive values assigned to each exercise of Test A.*

Exercise	Average Score	Difference Between Score and 50%	Distance in P.E. from Median	Distance Above Zero-point	Value Assigned
I	87.5	− 37.5	− 1.706	2.894	15
II	79.0	− 29.0	− 1.196	3.404	17
III	71.0	− 21.0	− 0.820	3.780	19
IV	53.5	− 3.5	− 0.130	4.470	23
V	34.1	+ 15.9	+ 0.608	5.208	26

Table VI gives the values assigned to each exercise of Test A and it indicates how this value was obtained. If we use the scale from 0 to 100 and have a normal distribution of pupils, the median pupil falls at 50. The first number in the column headed "Average score" shows that, when judged by exercise I alone, this median pupil will make a score of 87.5. That is

TABLE VII.—*Average positive scores for each exercise of Test B.*

Schools	Exercises				Number of Pupils
	I	II	III	IV	
XIV	67	55	47	32	19
XV	67	64	58	44	34
XVI	79	75	66	48	85
XVII	75	72	62	45	121
XVIII	82	78	70	50	267
XIX	83	78	76	52	344
XX	83	77	73	53	521
XXI	81	76	72	52	593
XXV	81	75	70	51	659
XXX	80	74	68	49	713
XXXI	80	73	66	48	766
XLVII	80	75	66	47	802
XLVIII	80	74	67	48	849
XLIX	81	75	67	48	935
LVII	81	75	67	47	975
LVIII	81	74	67	47	1025

exercise I is 87.3 − 50 or 37.3 too easy for the median pupil. Converting this into its P.E. value[1] we get − 1.706 which is

[1] For this purpose Table XIII of Trabue's Completion-Test Language Scale has been used.

2.894 P.E. above the assumed zero-point. The distances of the other exercises above the zero-point have been found in the same way. The values given in the last column of Table VI are proportional to these distances and they are such that their sum is 100.

TABLE VIII.—*Positive values assigned to each exercise of Test B.*

Exercise	Average Score	Difference Between Score and 50 %	Distance in P.E. from Median	Distance Above Zero-point	Value Assigned
I	80.7	− 30.7	− 1.286	3.314	21
II	74.4	− 24.4	− 0.972	3.628	23
III	66.5	− 16.5	− 0.632	3.968	26
IV	46.7	+ 3.3	+ 0.123	4.723	30

In a similar way, positive values have been assigned to the exercises of each of the other tests. The data and results are given in Tables VII–XIV. Tables VII, IX, XI and XIII show that a sufficient number of pupils has been tested to give fairly constant results for each test.

TABLE IX.—*Average positive scores for each exercise of Test C.*

Schools	Exercises				Number of Pupils
	I	II	III	IV	
VII	77	69	63	54	30
VIII	56	61	45	58	578
IX	55	60	45	57	651
X	55	59	44	58	682
XI	54	59	44	58	710
XII	53	57	42	57	795
XIII	53	57	41	57	844
XXVIII	53	56	40	55	882
XXXII	53	55	40	55	908
XXXIV	53	55	41	54	993
XXXVII	53	55	42	55	1019
XXXVIII	53	55	42	54	1047

TABLE X.—*Positive values assigned to each exercise of Test C.*

Exercise	Average Score	Difference Between Score and 50%	Distance in P.E. from Median	Distance Above Zero-point	Value Assigned
I	52.6	− 2.6	− 0.097	4.503	25
II	54.8	− 4.8	− 0.179	4.421	24
III	41.8	+ 8.2	+ 0.307	4.907	27
IV	54.4	− 4.4	− 0.164	4.436	24

TABLE XI.—*Average positive scores for each exercise of Test D.*

Schools	Exercises			Number of Pupils
	I	II	III	
V	88	68	65	88
XXII	85	70	63	132
XXIII	83	68	57	207
XXIV	86	70	54	275
XXV	83	71	51	350
XXVI	84	66	53	678
XXVII	86	71	54	827
XXIX	84	70	52	900
XLVI	83	68	52	1111

TABLE XII.—*Positive values assigned to each exercise of Test D.*

Exercise	Average Score	Difference Between Score and 50%	Distance in P.E. from Median	Distance Above Zero-point	Value Assigned
I	83.3	− 33.3	− 1.432	3.168	27
II	68.1	− 18.1	− 0.698	3.902	34
III	52.4	− 2.4	− 0.089	4.511	39

TABLE XIII.—*Average positive scores for each exercise of Test E.*

Schools	Exercises				Number of Pupils
	I	II	III	IV	
I	98	98	68	32	114
II	98	98	62	39	191
III	98	96	59	30	733
IV	98	96	57	31	773
V	98	96	58	36	865
VI	97	95	56	33	962
VII	98	95	55	34	992
LV	98	95	55	35	1036

TABLE XIV.—*Positive values assigned to each exercise of Test E.*

Exercises	Average Score	Difference Between Score and 50 %	Distance in P.E. from Median	Distance Above Zero-point	Value Assigned
I	97.7	− 47.7	− 2.958	1.642	12
II	94.9	− 44.9	− 2.425	2.175	16
III	54.6	− 4.6	− 0.172	4.428	33
IV	35.0	+ 15.0	+ 0.571	5.171	39

VI. CRITICAL EXAMINATION OF THE TESTS

Test D has been criticized on the ground that pupils will not understand what is to be done with the "Other known facts." The returns from this test show that such criticisms are not

well founded. In only a few cases did the pupils' papers show that they misunderstood the test. All such papers were rejected. Also some teachers have suggested that in Test E the lines drawn by the pupils are the results of guessing rather than thinking. While no doubt some pupils did guess, there is evidence that this is not generally true. In some cases the pupils drew new figures on their papers and tried out several lines before drawing the lines in the printed figure. Some of the pupils indicated the relation of parts of the figure by numbering the angles or marking the sides in some way. In many cases the pupils drew one or more incorrect lines in the figure, then erased them and drew the correct line. A careful examination of the papers showed that 56 per cent. of the pupils left some of these evidences of thought on their papers. Moreover many of those who drew the correct line and left no other evidence of their thought on their paper undoubtedly had a definite method of proof in mind.

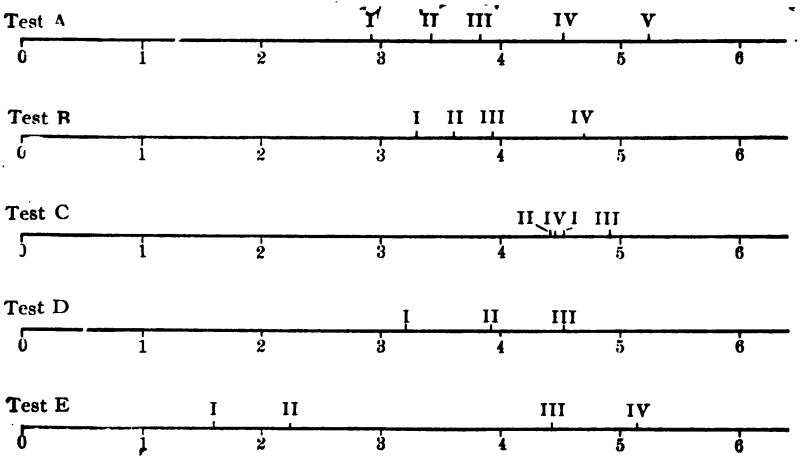


FIG. 2. Linear projection of the difficulty of the exercises as shown in Tables VI, VIII, X, XII, XIV.

The time required for the solution of a single exercise has made it impossible to include a large number of exercises in each test. A larger number of carefully selected exercises would, undoubtedly, make possible a more accurate discrimination between varying degrees of ability. However, this difficulty is not as great as it at first appears; for, with the exception of

Test E, each exercise consists of a number of steps each of which may be considered as a separate exercise, just as the words in a sentence may be used as separate elements in a spelling test. The real difficulty here lies in the fact that we have not been able to evaluate these steps separately.[1]

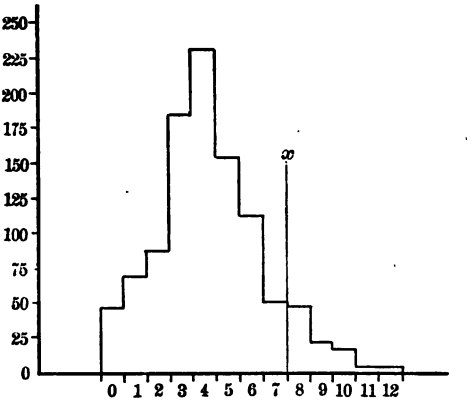


FIG. 3. Distribution given by Exercise I, Test C.
Note.—The vertical line x marks off approximately the upper ten per cent. of the class.

Figure 2 is the linear projection of the difficulties of the exercises as shown under "Distances above zero-point" in Tables VI, VIII, X, XII, XIV. The exercises of the different tests do not begin at, or extend to, the same points on the scale. Nor are they distributed in the same manner over the portion of the scale which they do occupy. Hence the tests will not measure

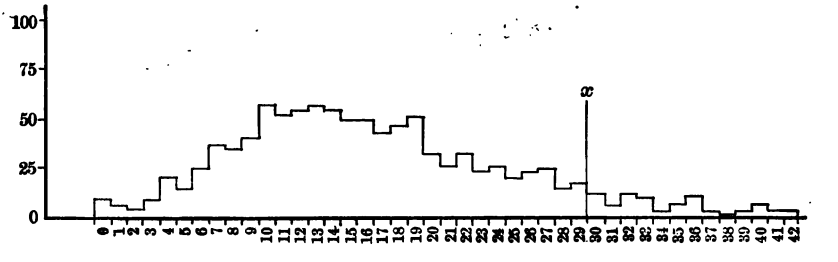


FIG. 4. Distribution given by Exercise II, Test C.
Note.—The vertical line x marks off approximately the upper ten per cent. of the class.

[1] Page 44.

the respective abilities in the same manner, and therefore the results obtained from the different tests can not be compared. Also the tests would be more satisfactory if the exercises were

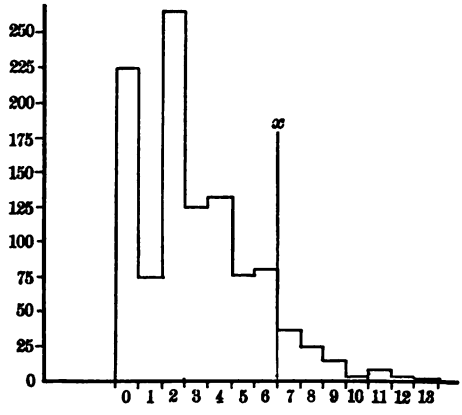


FIG. 5. Distribution given by Exercise III, Test C.
Note.—The vertical line x marks off approximately the upper ten per cent. of the class.

distributed over a larger portion of the scale and separated by more nearly equal intervals. In this respect Test C demands special attention. The exercises are apparently of almost the

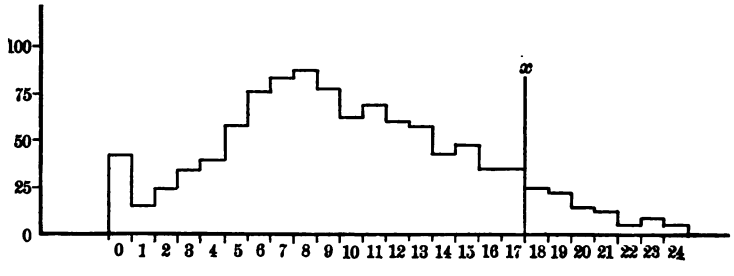


FIG. 6. Distribution given by Exercise IV, Test C.
Note.—The vertical line x marks off approximately the upper ten per cent. of the class.

same degree of difficulty. This is due, however, to the arbitrary way in which the number of facts considered as a perfect answer was selected.[1] If the separate exercises gave exactly the same

[1] Page 36.

distribution of pupils and we had considered as a perfect answer to each exercise the smallest number of facts given correctly by any one of exactly the highest ten per cent. of pupils taking the test, the different exercises would have presented the same degree of difficulty. However, as Figs. 3 to 6 show, the separate exer-

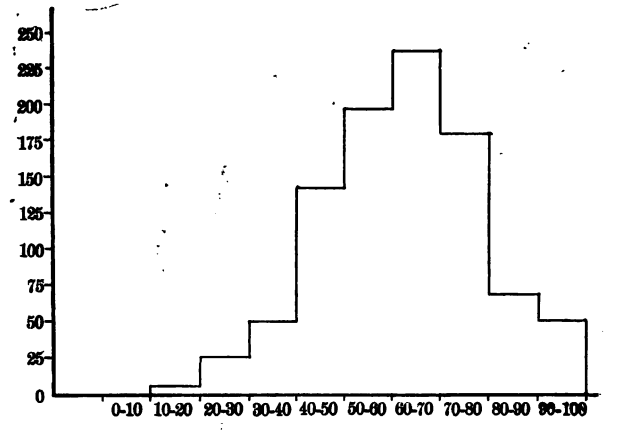


FIG. 7. Distribution given by the positive scores of Test A.

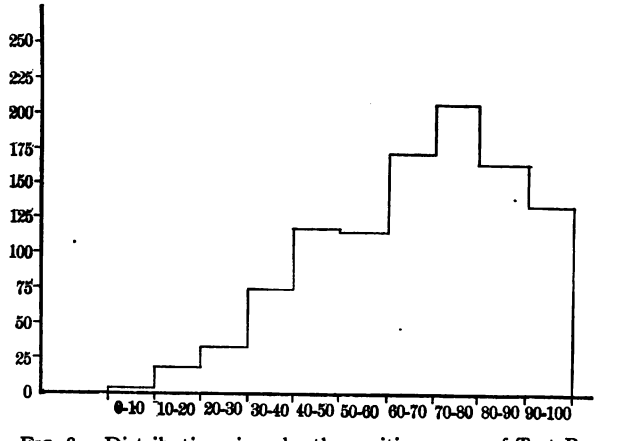


FIG. 8. Distribution given by the positive scores of Test B.

cises did not give the same distribution, and, as previously noted,[1] we were not able to select exactly the highest ten per cent. of the pupils. Hence there is some variation in the amount of difficulty presented by the different exercises of this test, but that variation is slight.

[1] Page 36.

In any test an important consideration is the form of distribution which it gives. Figures 7 to 11 represent the data given in Tables[1] XXVIII to XXXII and give the distribution according to the positive scores for Tests A, B, C, D and E[2] respectively.

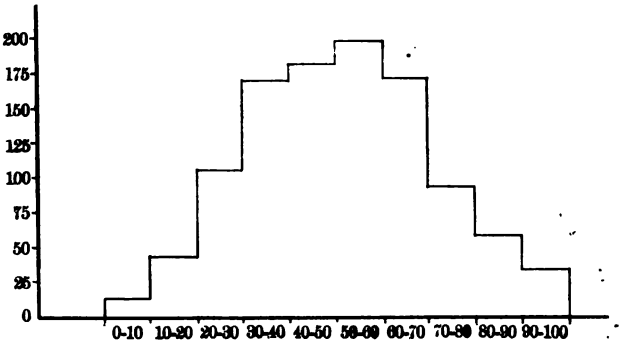


FIG. 9. Distribution given by the positive scores of Test C.

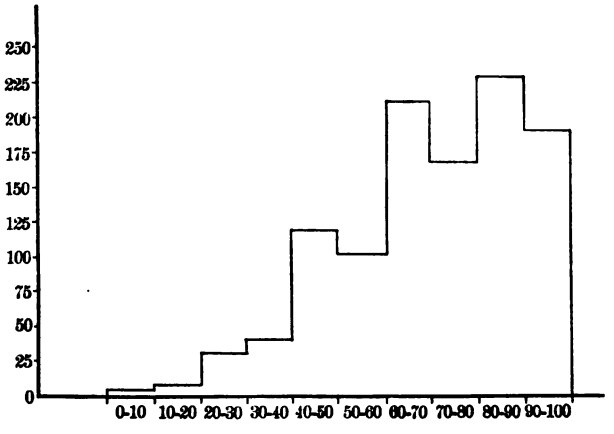


FIG. 10. Distribution given by the positive scores of Test D.

The curves for Tests A and C are nearly normal, but those for Tests B, D, and E are badly skewed toward the high end of the scale. Figures 12 to 16 are the frequency curves for the negative scores and represent the data of Tables[3] XXXIII to XXXVII.

[1] Pages 75–88.

[2] As Test E grouped the pupils at only a few points of the scale the class-interval in Fig. 11 has been made twice as large as in Figs. 7–10.

[3] Pages 89–93.

With the exception of the curve for Test A, they also are badly skewed toward the high end of the scale.

This skewness may be due in part to the elimination of the poorer pupils throughout the elementary school and the first year of the high school, giving a specially selected group with

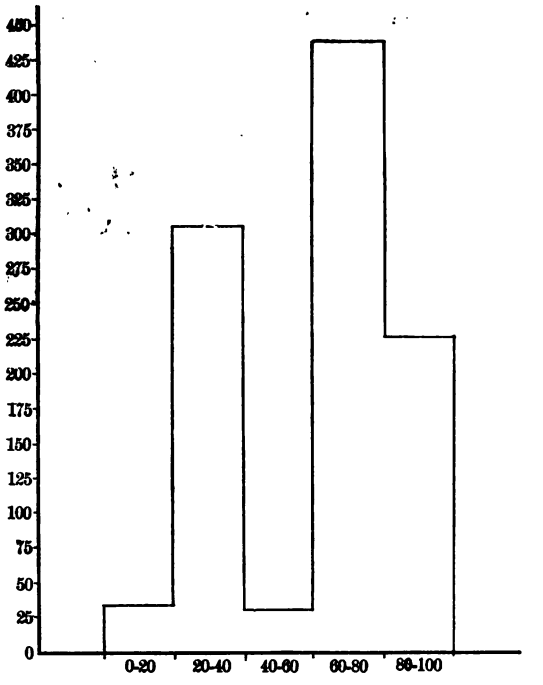


FIG. 11. Distribution given by the positive scores of Test E.

which we have worked. The importance of this factor is increased in those schools in which there had been a promotion between the time the study of geometry was begun and the time the tests were given, resulting in an elimination of pupils who had begun the study.[1]

However, a more important cause of the skewness is the selection of the exercises. Tests B, D and E are somewhat too easy. Also there are too few exercises in Test E and, as we have seen, they are not distributed at equal intervals along the scale. So far as the positive scores of this test are concerned the

[1] Page 19.

exercises do not admit of partial answers. Hence, with only four exercises not more than fifteen different scores are possible. In fact Table XXXIII shows that the exercises are such that the pupils are, for the most part, grouped at four points of the scale.

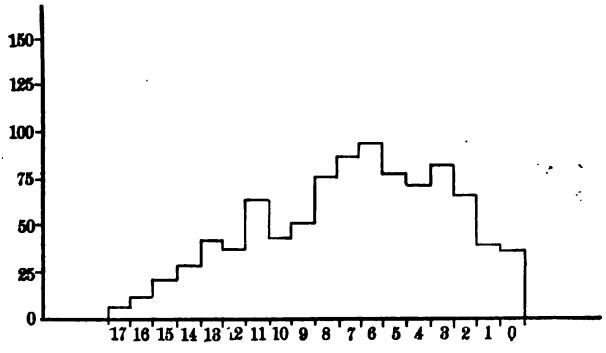


FIG. 12. Distribution given by the negative scores of Test A.

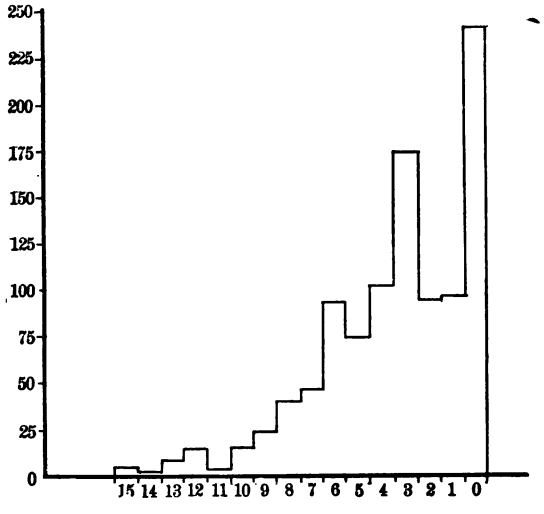


FIG. 13. Distribution given by the negative scores of Test B.

Evidently two exercises selected so as to fall at equal intervals between exercises II and III would give more satisfactory results. Some of the pupils who answered exercises I and II but could not answer III or IV would be able to answer one or both of these new exercises. Also some of those who answered either one or both of III and IV would fail to answer one of the new

exercises. Hence more pupils would be grouped along the central portion of the scale and fewer at the ends. The exercises of the other tests admit of partial answers and therefore this same

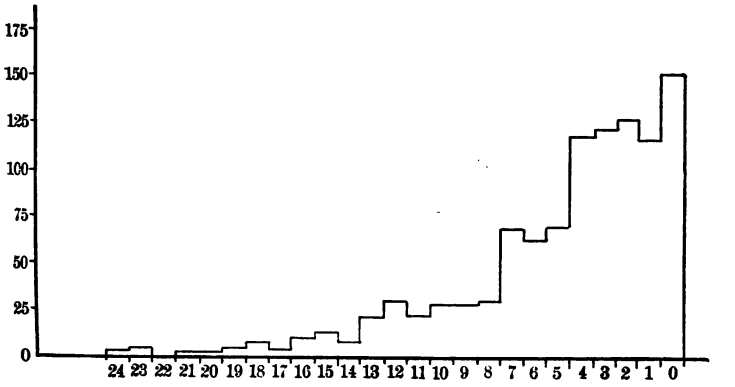


FIG. 14. Distribution given by the negative scores of Test C.

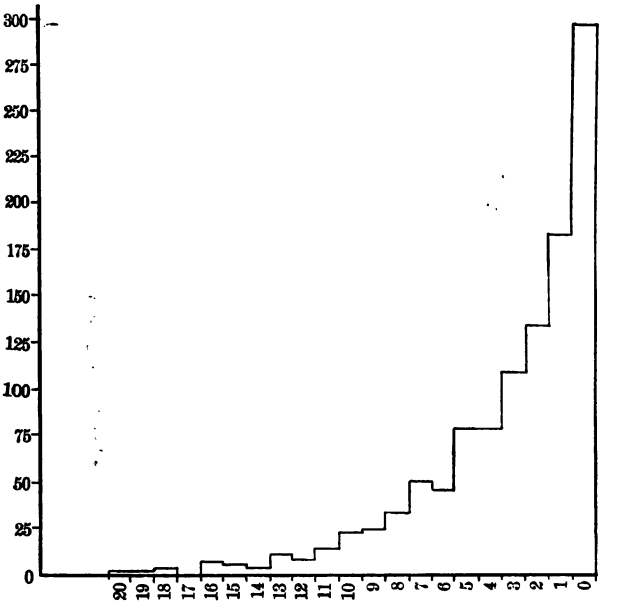


FIG. 15. Distribution given by the negative scores of Test D.

difficulty does not arise in connection with them. Tables XXVII to XXXI show that there is no pronounced tendency to group pupils at a few points of the scale.

When considering the skewed distributions we must remember that neither the positive nor negative score is complete in itself. If these scores could be combined the curve would be moved toward the lower end of the scale and the skewness would be decreased. That is, if a pupil's positive and negative scores could be combined, the result would be the same as the positive

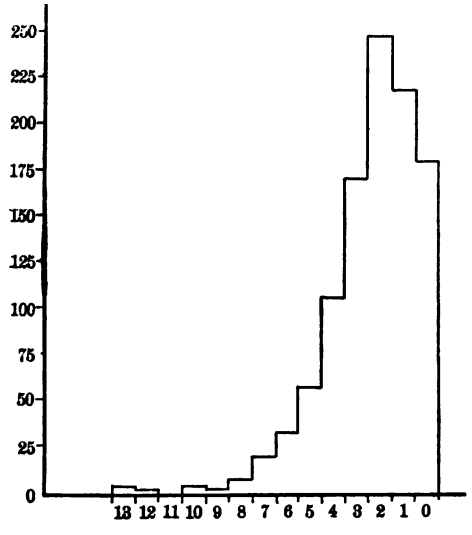


FIG. 16. Distribution given by the negative scores of Test E.

score only when the negative score is zero. In all other cases it would be less and he would take a lower position on the scale. It may be argued that those pupils who made a high positive score will make a negative score near zero and conversely, leaving their position on the scale practically unchanged. This, however, has been shown not to be the case.[1]

Furthermore, in this study we are not so much concerned with the exact measure of pupils' abilities as we are with their ranks when arrayed according to their abilities. If our tests enable us to say that one pupil is better than another without saying how much better, our purpose will be served. That this condition is satisfied by Tests A, B, C and D is indicated in Tables XXVIII to XXXI by the fact that there is not a strong tendency to group pupils around a few points of the scale. Hence

[1] Pages 24–27.

we may conclude that, for our purpose, Tests A, B, C and D are fairly satisfactory, but that Test E is not. The data of Test E will be included in this study to throw whatever light it may on our conclusions and to indicate a possible field for further investigation.[1]

VII. Examination of School Grades

Since our purpose is to compare the results of the tests with the pupils' school grades we shall investigate the reliability of

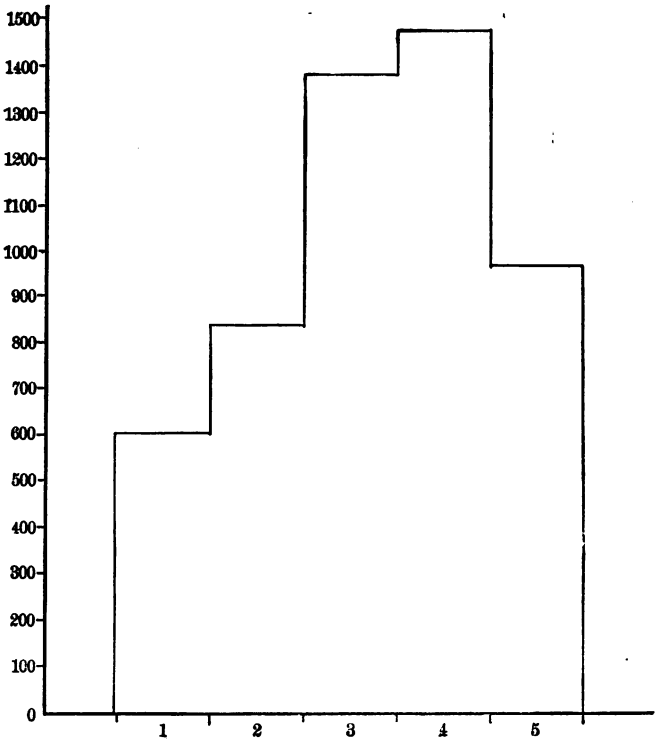


Fig. 17. Distribution of 5195 pupils given by their school grades.

the school grades as shown by the distribution which they give. There is a great variation in the part of the scale used by different schools. One school may give grades between 65 and 100 while another gives them between 40 and 80. Although the validity

[1] The author plans to improve Test E and make further investigation with it in the near future.

TABLE XV.—*Number of pupils falling within each interval into which the part of the scale used by each school is divided.*

Schools	1	2	3	4	5	Totals
I	22	46	14	27	5	114
II	7	24	16	21	9	77
III	4	12	130	277	119	542
IV	2	4	5	17	12	40
V	7	5	29	29	22	92
VI	14	39	0	30	14	97
VII	3	5	12	7	3	30
VIII	8	94	245	103	98	548
IX	21	22	11	16	3	73
X	5	4	9	9	4	31
XI	6	3	12	6	1	28
XII	2	1	24	35	23	85
XIII	16	9	11	6	7	49
XIV	1	5	5	5	3	19
XV	2	3	1	8	1	15
XVI	27	3	12	5	4	51
XVII	1	7	9	11	8	36
XVIII	4	3	22	55	62	146
XIX	29	21	7	7	13	77
XX	1	3	93	54	26	177
XXI	1	17	37	0	17	72
XXII	6	12	17	7	2	44
XXIII	5	21	49	57	31	163
XXIV	3	3	10	27	25	68
XXV	24	37	35	62	55	213
XXVI	196	68	23	34	7	328
XXVII	1	33	68	30	17	149
XXVIII	4	6	8	11	9	38
XXIX	2	12	39	15	5	73
XXX	2	3	8	28	13	54
XXXI	6	11	28	7	1	53
XXXII	1	5	0	16	4	26
XXXIII	4	0	8	0	3	15
XXXIV	3	12	33	21	16	85
XXXV	1	2	6	10	4	23
XXXVI	3	2	3	11	7	26
XXXVII	1	3	4	9	9	26
XXXVIII	11	4	6	2	5	28
XXXIX	6	7	0	3	1	17
XL	6	18	12	7	4	47
XLI	2	6	13	11	16	48
XLII	6	10	14	13	8	51
XLIII	1	4	8	19	21	53
XLIV	5	8	15	7	17	52
XLV	6	7	8	13	17	51

TABLE XV.—*Continued.*

Schools	1	2	3	4	5	Totals
XLVI	14	56	43	59	39	211
XLVII	1	7	9	13	6	36
XLVIII	2	9	13	12	11	47
XLIX	4	15	21	27	19	86
L	1	2	3	5	3	14
LI	3	2	3	4	2	14
LII	2	6	11	22	7	48
LIII	3	6	18	27	17	71
LIV	1	0	9	6	6	22
LV	1	3	12	9	19	44
LVI	6	6	9	23	19	63
LVII	2	6	9	13	8	38
LVIII	7	11	17	9	8	52
LIX	1	4	5	14	5	29
LX	3	5	5	10	6	29
LXI	10	3	2	3	7	25
LXII	43	39	34	27	7	150
LXIII	6	15	23	30	14	88
Totals.....	596	819	1365	1461	954	5195

of a grading system does not depend as much upon the part of the scale used as it does on the accuracy of the distribution along that part; nevertheless, in order to compare different

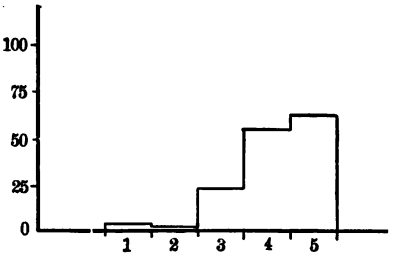


FIG. 18. Distribution of the 146 pupils of school XVIII given by school grades.

grading systems and to combine the grades from different schools it is necessary to eliminate, so far as is possible, any such variation. In order to do this the part of the scale used by each school has been divided into five equal intervals and the pupils have been grouped according to the intervals within which they fall. Beginning with the lowest the intervals are numbered from 1 to 5.

Table XV shows how the 5,195[1] different pupils tested are distributed according to their school grades. The distribution of the total number of pupils is represented graphically in Fig. 17.

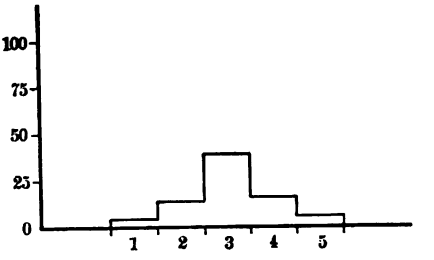


FIG. 19. Distribution of the 73 pupils of school XXIX given by school grades.

While this roughly approximates a normal frequency surface, there is a decided skewness towards the higher end of the scale. This may be due in part to the elimination of the poorer pupils throughout the elementary school and the first year of the high school, but it no doubt also indicates a tendency on the

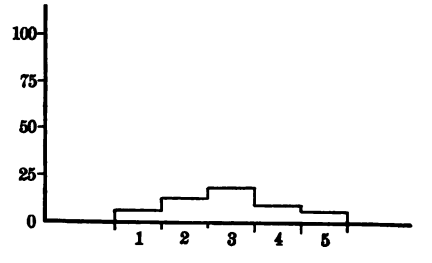


FIG. 20. Distribution of the 52 pupils of school LVIII given by school grades.

part of teachers to avoid not only the lower end of the scale, but also the lower end of that portion of the scale which they use. A study of Table XV reveals a decided variation in the form of distribution given by the grades of the different schools. Figures 18 to 23 represent separately the distributions of the pupils of six schools. Figure 18 is decidedly skewed towards the higher end of the scale while Fig. 21 is skewed towards the lower

[1] The sum of the totals given in Tables V, VII, IX, XI, XIII and the 150 pupils from school LXII not included in these totals is 5,313. This apparent inconsistency is due to the fact that 88 pupils of school V and 30 pupils of school VII took two tests, making 118 duplications in the total 5,313 pupils.

end. Undoubtedly these distributions are not based accurately on the pupils' abilities. If we were to draw the frequency surfaces

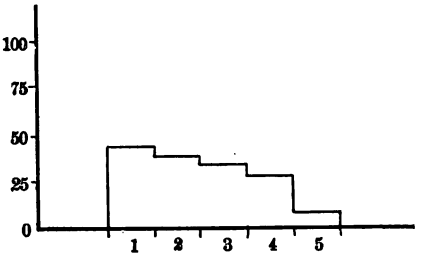


FIG. 21. Distribution of the 150 pupils of school LXII given by school grades.

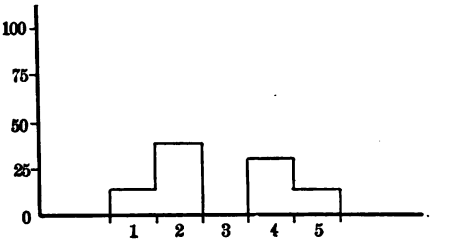


FIG. 22. Distribution of the 97 pupils of school VI given by the school grades.

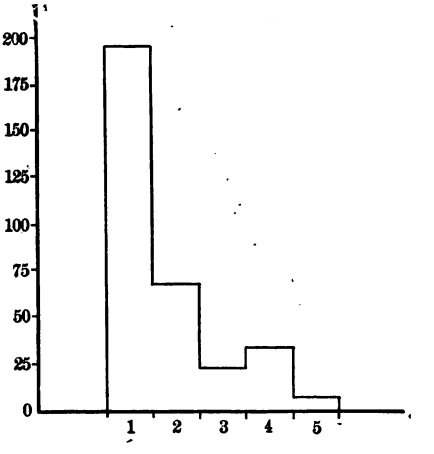


FIG. 23. Distribution of the 328 pupils of school XXVI given by school grades.

for each of the sixty-three schools given in Table XV, we would find various forms of distribution lying between these two extremes. Some of these are approximately normal as is shown

by Figs. 19 and 20. Others represent peculiar irregularities. Figure 22 gives a distribution of 97 pupils in which no case falls within the middle interval, a condition which could scarcely exist if this number of pupils were distributed accurately according to their abilities. Figure 23 gives even a more strikingly irregular distribution. In this case 60 per cent. of the pupils fall within the lowest interval. In fact 74 of the 328 pupils, or more than 22 per cent., received the lowest passing mark.

Thus it is evident that, even where a fairly large group of pupils is involved, the school grades frequently do not give a normal distribution and they very probably are not reliable measures of the pupils' abilities. We shall now investigate the relation of the test and school grades.

VIII. Comparison of School and Test Grades

As noted on page 7 a teacher's grades *should* be a measure of those abilities which she considers of value. This, of course, will not always be the case. No doubt these grades are often a measure of the pupil's ability to memorize and reproduce a page of geometry text, although the teacher would scarcely admit that such is the case. If the abilities with which this study is concerned are among those which the teacher considers of value, then there should be a positive correlation between the test and the school grades. This correlation should not be perfect for there are several abilities involved in the study of geometry and there should not be a perfect correspondence between grades based on a number of abilities and those based on only one of these abilities. To the degree that the tests measure the four abilities in question, the coefficients of correlation will be an index of one of two things; namely, the extent to which the teacher considers these abilities of value, or the extent to which she has been able to base her grades on the abilities which she believes to be of value.

Method of Determining the Correlation.—Since it has been impossible to combine the positive and negative scores for the different tests, it is impossible to compute the coefficient of correlation by a method which requires actual measures of abilities. If, however, the individuals can be arranged in a series according to one trait and then rearranged according to a second trait the

correlation between the two traits may be obtained by the formula

$$\rho = 1 - \frac{6\Sigma d^2}{N(N^2 - 1)}$$

where ρ is the coefficient of correlation, d is the difference between an individual's rank in one series and his rank in the other series, and N is the number of individuals considered.[1] However, this formula assumes the difference between any two successive ranks to be the same for all parts of the scale, an assumption which is not true. To correct this error Professor K. Pearson has developed the formula,

$$r = 2 \sin\left(\frac{\pi}{6}\rho\right)$$

where

$$\rho = 1 - \frac{6\Sigma d^2}{N(N^2 - 1)}$$

and r is the true coefficient of correlation. The probable error is given by the formula,

$$\text{P.E.} = \frac{0.7063(1 - r^2)}{\sqrt{N}}.$$

This means that it is an even chance that the true coefficient of correlation falls within the limits $r \pm$ P.E. and the chances are 16 to 1 that the true coefficient will fall within the limits $r \pm 3$P.E. That is, we may be fairly sure that there is a positive correlation when $r > 3$P.E. This method of computing the coefficient of correlation will be suited to our purpose if we can rank the pupils according to the school grades and again according to the test scores. But we encounter a difficulty here from the fact that each pupil has two test scores. The following plan has been adopted in all cases. The pupils of each school were arranged according to their positive test scores. Beginning with the poorest pupil, they were numbered from 1 upward.[2] If two or more pupils tied for the same places, the sum of the numbers belonging to these places was distributed equally among them. In a similar way numbers were assigned according to the negative scores, the higher number always being assigned to the better pupil. The two numbers thus assigned to each pupil were then

[1] For a discussion of this method of computing the coefficient of correlation see William Brown, "The Essentials of Mental Measurement," pp. 42–53.

[2] It is to be noted that this reverses the usual order in which ranks are assigned.

added, and the pupils were again ranked according to these sums. These final ranks were used to compute the coefficient of correlation between the school and the test marks. This gives equal weight to the positive and negative scores, and we are not able to prove that this is as it should be. However, it is probably as accurate as any other method of combining the two elements.

Method of Dealing with the Data from Different Schools.—If the data from the various schools could be combined for each test we would gain the advantage of a single measure of correlation for that test. However, this would cause us to lose sight of the peculiarities of individual schools. Moreover such a procedure is impossible because of the great variation in the grading systems of the various schools. A pupil marked 75 in one school may be a better student than one marked 90 in a second school. Unless there is some means of reducing these grades to the same basis it would be impossible to arrange the pupils of the different schools in a single series according to their school grades. Hence in this study the schools have been dealt with separately. The several coefficients of correlation will indicate the general tendency in a cumulative way and, at the same time, reveal the differences in the practices of the several schools. It may be argued that there is also a variation in the grading systems of individual teachers. This is true, but usually the variation is not so great in the case of teachers in the same system as it is in the case of different schools. Constant intercourse among the teachers and other influences within the school tend to unify the standards of a department. However, the fact remains that there is a variation in the teachers' standards and this to a certain extent weakens our conclusions.

The Coefficients of Correlation.—Space does not permit of a complete statement of the computation of the coefficient of correlation for each of the sixty-three schools. The work for one school is given in detail below. Only the results are given for the other schools.

The pupils of school XXXVI took Test A. The differences between their ranks according to their test and school grades and the method of obtaining these differences are given in Table XVI.[1] In Table XVII, d is the difference between test

[1] The numbers of the first column replace the pupils' names and have no relation to their ranks.

TABLE XVI.—*Differences between ranks according to test and school scores.*

Pupil	Rank According to + Scores	Rank According to − Scores	Sum of + and − Ranks	Rank According to Sum of + and − Ranks	Rank According to School Grades	Difference Between Test and School Rank
1	14.5	13.5	28.0	13.0	24.5	11.5
2	5.0	8.0	13.0	4.0	17.5	13.5
3	6.5	13.5	20.0	7.5	17.5	10.0
4	1.0	5.0	6.0	2.0	8.0	6.0
5	16.0	2.0	18.0	5.0	22.0	17.0
6	24.0	22.0	46.0	25.0	22.0	3.0
7	23.0	9.5	32.5	19.0	17.5	1.5
8	26.0	22.0	48.0	26.0	24.5	1.5
9	8.5	22.0	30.5	17.5	14.0	3.5
10	25.0	18.5	43.5	24.0	26.0	2.0
11	20.0	13.5	33.5	20.0	22.0	2.0
12	20.0	9.5	29.5	16.0	14.0	2.0
13	20.0	22.0	42.0	23.0	6.0	17.0
14	10.5	18.5	29.0	14.5	4.5	10.0
15	22.0	3.0	25.0	11.0	14.0	3.0
16	18.0	17.0	35.0	21.0	2.5	18.5
17	2.0	1.0	3.0	1.0	2.5	1.5
18	10.5	13.5	24.0	9.5	7.0	2.5
19	17.0	7.0	24.0	9.5	9.5	0.0
20	3.0	26.0	29.0	14.5	1.0	13.5
21	13.0	25.0	38.0	22.0	9.5	12.5
22	8.5	22.0	30.5	17.5	11.5	6.0
23	4.0	5.0	9.0	3.0	4.5	1.5
24	14.5	5.0	19.5	6.0	11.5	5.5
25	12.0	13.5	25.5	12.0	20.0	8.0
26	6.5	13.5	20.0	7.5	17.5	10.0

and school ranks, K is the number of times each difference occurs in Table XVI, and Σd^2 is the sum of the squares of the differences.

TABLE XVII.—*Sum of the squares of the differences between test and school ranks.*

d	K	Kd^2	d	K	Kd^2
0.0	1	0.00	8.0	1	64.00
1.5	4	9.00	10.0	3	300.00
2.0	3	12.00	11.5	1	132.25
2.5	1	6.25	12.5	1	156.25
3.0	2	18.00	13.5	2	364.50
3.5	1	12.25	17.0	2	578.00
5.5	1	30.25	18.5	1	342.25
6.0	2	72.00			
					$\Sigma d^2 = 2097.00$

Substituting $N = 26$ and $\Sigma d^2 = 2097$ in the formula

$$\rho = 1 - \frac{6\Sigma d^2}{N(N^2 - 1)}$$

we get

$$\rho = 0.28.$$

Correcting this result by the formula

$$r = 2 \sin\left(\frac{\pi}{6}\rho\right)$$

we get

$$r = 0.292.$$

The formula

$$\text{P.E.} = \frac{0.7063(1 - r^2)}{\sqrt{N}}$$

gives

$$\text{P.E.} = 0.127.$$

The coefficient is less than three times the probable error. Hence in the case of school XXXVI the pupils' ability to draw a figure for a theorem as measured by Test A has but slight, if any, relation to the school grades which they received.

TABLE XVIII.—*Coefficients of correlation for Test A.*

School	r	P.E.	Relation of r to 3 P.E.
XXIII	0.313	0.068	$r > 3$ P.E.
XXV	0.395	0.071	$r > 3$ P.E.
XXXIII	0.111	0.180	$r < 3$ P.E.
XXXV	0.628	0.089	$r > 3$ P.E.
XXXVI	0.303	0.125	$r < 3$ P.E.
XXXIX	0.426	0.140	$r > 3$ P.E.
XL	0.487	0.079	$r > 3$ P.E.
XLI	0.303	0.093	$r > 3$ P.E.
XLII	0.697	0.051	$r > 3$ P.E.
XLIII	0.436	0.088	$r > 3$ P.E.
XLIV	0.364	0.085	$r > 3$ P.E.
XLV	0.364	0.086	$r > 3$ P.E.
L	0.588	0.123	$r > 3$ P.E.
LI	0.240	0.188	$r < 3$ P.E.
LII	0.528	0.074	$r > 3$ P.E.
LIII	0.588	0.055	$r > 3$ P.E.
LIV	−0.150	0.147	$r < 3$ P.E.
LVI	0.199	0.085	$r < 3$ P.E.
LIX	0.688	0.069	$r > 3$ P.E.
LX	0.578	0.087	$r > 3$ P.E.
LXI	0.548	0.099	$r > 3$ P.E.
LXII	0.436	0.053	$r > 3$ P.E.
LXIII	0.292	0.069	$r > 3$ P.E.

A study of Table XVIII shows that for Test A the coefficient of correlation varies from − 0.150 to 0.697. For 18 of the 23 schools the coefficient is greater than 3P.E. and therefore has scientific significance. For schools XXXV, XLII, L, LII, LIII, LIX, LX and LXI the coefficients are probably as large as we can expect, if we remember that the ability to draw a figure is only one of several factors upon which school grades in geometry depend. In schools XXXIII, XXXVI, LI, LIV and LVI there seems to be but little relation between school grades and the ability to draw figures. In the remaining schools the positive correlations are but slight. In school LXII the test was given after all of plane geometry had been completed and the results were compared with school grades given for the first two books. We would expect such a condition to reduce the coefficient of correlation. Nevertheless Table XVIII shows that there was considerable relation between the test and school grades. If data were at hand, it would be interesting to determine the effect of the second half-year of training on the rank of pupils as determined by the first half-year of training.

TABLE XIX.—*Coefficients of correlation for Test B.*

School	r	P.E.	Relation of r to 3 P.E.
XIV	0.467	0.127	$r > 3$ P.E.
XV	0.548	0.127	$r > 3$ P.E.
XVI	0.447	0.079	$r > 3$ P.E.
XVII	0.140	0.115	$r < 3$ P.E.
XVIII	0.188	0.056	$r > 3$ P.E.
XIX	0.230	0.076	$r > 3$ P.E.
XX	0.219	0.051	$r > 3$ P.E.
XXI	0.323	0.075	$r > 3$ P.E.
XXV	0.568	0.059	$r > 3$ P.E.
XXX	0.395	0.081	$r > 3$ P.E.
XXXI	0.271	0.090	$r > 3$ P.E.
XLVII	0.668	0.065	$r > 3$ P.E.
XLVIII	0.261	0.096	$r < 3$ P.E.
XLIX	0.538	0.054	$r > 3$ P.E.
LVII	0.588	0.075	$r > 3$ P.E.
LVIII	0.344	0.086	$r > 3$ P.E.

Table XIX shows that the coefficients of correlation between the scores for Test B and the school grades are generally low. For 14 of the 16 schools tested the coefficients are greater than

3 P.E., and therefore it is quite probable that there is a positive correlation in these cases. For schools XV, XXV, XLVII, XLIX and LVII the coefficients are probably as large as can be expected, but for the other schools the correlation is low and in schools XVIII and XLVIII there is, perhaps, little or no relation between the pupil's ability to state the hypothesis and conclusion and the ability upon which his school grade is based.

TABLE XX.—*Coefficients of correlation for Test C.*

School	r	P.E.	Relation of r to 3 P.E.
VII	0.209	0.123	$r < 3$ P.E.
VIII	0.548	0.030	$r > 3$ P.E.
IX	0.178	0.080	$r < 3$ P.E.
X	0.385	0.108	$r > 3$ P.E.
XI	0.416	0.110	$r > 3$ P.E.
XII	0.538	0.054	$r > 3$ P.E.
XIII	0.333	0.090	$r > 3$ P.E.
XXVIII	0.436	0.093	$r > 3$ P.E.
XXXII	0.042	0.138	$r < 3$ P.E.
XXXIV	0.487	0.058	$r > 3$ P.E.
XXXVII	0.406	0.116	$r > 3$ P.E.
XXXVIII	0.508	0.099	$r > 3$ P.E.

Table XX shows that the correlation between the scores for Test C and the school grades is generally low. For 9 of the 12 schools tested the coefficients of correlation are greater than 3 P.E. and therefore there is very probably a positive correlation between the pupil's ability to recall facts about a figure and his school grade. For schools VIII, XII and XXXVIII the coefficients are probably as large as can be expected, but for the other schools they are low and in schools VII, IX and XXXII there is, perhaps, little or no relation between a pupil's ability to recall geometrical facts and his school grades.

Table XXI shows that a similar condition exists for Test D. There is generally a low positive correlation between the pupil's ability to select and arrange facts to produce a proof and his school grade. Of the 9 schools tested 7 have a coefficient of correlation greater than 3 P.E. For schools XXII and XXIX the coefficients are probably as large as can be expected, but for the other schools they are small and in schools XXIII and XXVII there is, perhaps, little or no relation between the test and school grades.

Although the selection of exercises for Test E is far from satisfactory, Table XXII shows almost as favorable results as were obtained from the other tests. There is generally a low positive correlation between the test and school grades. Six of the eight schools tested have a coefficient greater than 3 P.E. and therefore there is very probably a positive correlation between the ability to draw auxiliary lines and the abilities upon which school grades are based. The coefficient for school II is fairly large, but for the other schools it is usually low, and in schools

TABLE XXI.—*Coefficients of correlation for Test D.*

School	r	P.E.	Relation of r to 3 P.E.
V	0.351	0.066	$r > 3$ P.E.
XXII	0.528	0.077	$r > 3$ P.E.
XXIII	0.209	0.078	$r < 3$ P.E.
XXIV	0.325	0.077	$r > 3$ P.E.
XXV	0.395	0.013	$r > 3$ P.E.
XXVI	0.303	0.035	$r > 3$ P.E.
XXVII	0.126	0.057	$r < 3$ P.E.
XXIX	0.568	0.056	$r > 3$ P.E.
XLVI	0.323	0.044	$r > 3$ P.E.

TABLE XXII.—*Coefficients of correlation for Test E.*

School	r	P.E.	Relation of r to 3 P.E.
I	0.216	0.060	$r > 3$ P.E.
II	0.608	0.051	$r > 3$ P.E.
III	0.139	0.029	$r > 3$ P.E.
IV	0.031	0.112	$r < 3$ P.E.
V	0.493	0.056	$r > 3$ P.E.
VI	0.253	0.067	$r > 3$ P.E.
VII	0.229	0.122	$r < 3$ P.E.
LV	0.343	0.094	$r > 3$ P.E.

IV and VII there is, perhaps, little or no relation between the test and school grades. The comparatively favorable results obtained from the poorly selected exercises of Test E may be due to the fact that these exercises test several abilities rather than a single ability. That is, if it is true that a pupil must have a definite proof of a theorem in mind before he can draw the proper auxiliary lines, then Test E will measure the same abilities that Tests C and D measure and therefore, other things being equal, Test E should give the highest correlation.

Conclusion.—Among the different schools there is a great variation in the relation between the test and school grades. There is usually a positive correlation but in only a few schools is this correlation high. In some of the schools the coefficient is, perhaps, affected by the elimination of the poorer pupils.[1] As it is easier to distinguish the extreme cases, the elimination of the poorer pupils would tend to reduce the correlation. But if a correction could be made for this, it is quite probable that the correlation would remain low.

Most schools, in some way, emphasize each of the four abilities which this study investigates. If these abilities are of value in themselves or if they furnish a basis for other results which are of value, the school grades should bear a closer relation to them. If, on the other hand, the coefficients of correlation can be taken as indices of the values of these abilities, then these values are, in many cases, so slight that the schools are scarcely justified in giving as much time to this phase of geometry as is now given to it.

IX. The Extent to which the Abilities are Developed

Constancy of Results.—It will be of interest to see the extent to which the schools succeed in developing each of the four abilities. For this purpose the median scores for the pupils of each school and for all the pupils have been computed. However, before drawing any conclusions from the combined data of the different schools we should determine whether a sufficient number of pupils has been tested to eliminate chance variation. As previously noted Tables V, VII, IX, XI and XIII show that, so far as positive scores are concerned, this condition has been fairly well realized. Tables XXIII to XXVII give the average negative scores for each test in a cumulative way. A study of these tables shows that a fairly constant condition has been obtained in the case of each test. The results of Test D are less satisfactory in this respect than those of any of the other tests.

[1] Page 19.

TABLE XXIII.—*Average negative scores for each exercise of Test A.*

Schools	Exercises					Number of Pupils
	I	II	III	IV	V	
XXIII	0.56	0.54	0.49	3.84	3.07	88
XXV	0.66	0.52	0.57	3.10	1.15	158
XXXIII	0.64	0.54	0.60	3.01	1.88	173
XXXV	0.63	0.54	0.64	2.91	2.06	196
XXXVI	0.60	0.54	0.70	2.73	2.12	222
XXXIX	0.62	0.53	0.64	2.62	2.10	239
XL	0.59	0.51	0.71	2.76	2.10	286
XLI	0.53	0.49	0.68	2.74	2.02	334
XLII	0.49	0.45	0.68	2.60	2.12	385
XLIII	0.46	0.42	0.65	3.00	2.11	438
XLIV	0.44	0.41	0.66	2.88	2.13	490
XLV	0.41	0.39	0.63	3.04	2.25	541
L	0.41	0.39	0.63	3.03	2.30	555
LI	0.41	0.38	0.64	3.02	2.30	569
LII	0.41	0.40	0.66	2.87	2.27	617
LIII	0.43	0.41	0.70	3.12	2.38	688
LIV	0.45	0.41	0.70	3.09	2.35	710
LVI	0.45	0.44	0.71	3.02	2.34	773
LIX	0.45	0.44	0.69	3.01	2.36	802
LX	0.45	0.44	0.68	3.12	2.39	831
LXI	0.44	0.44	0.68	3.07	2.30	856
LXIII	0.44	0.43	0.71	2.96	2.39	944

TABLE XXIV.—*Average negative scores for each exercise of Test B.*

Schools	Exercises				Number of Pupils
	I	II	III	IV	
XIV	0.16	0.32	1.83	2.63	19
XV	0.24	0.29	1.32	1.97	34
XVI	0.16	0.21	1.12	1.93	85
XVII	0.14	0.18	0.94	1.92	121
XVIII	0.08	0.18	0.75	2.24	267
XIX	0.07	0.17	0.68	2.13	344
XX	0.09	0.25	0.71	2.16	521
XXI	0.10	0.24	0.74	2.13	593
XXV	0.12	0.25	0.80	2.17	659
XXX	0.13	0.28	0.84	2.15	713
XXXI	0.14	0.28	0.81	2.17	766
XLVII	0.14	0.28	0.90	2.14	802
XLVIII	0.13	0.27	0.89	2.16	849
XLIX	0.13	0.27	0.90	2.15	935
LVII	0.13	0.27	0.90	2.16	975
LVIII	0.14	0.28	0.92	2.16	1025

TABLE XXV.—*Average negative scores for each exercise of Test C.*

Schools	Exercises				Number of Pupils
	I	II	III	IV	
VII	0.23	0.23	1.77	1.93	30
VIII	0.28	0.58	1.84	2.02	578
IX	0.26	0.59	1.87	2.08	651
X	0.26	0.58	1.87	2.09	682
XI	0.26	0.58	1.84	2.10	710
XII	0.28	0.63	1.81	2.09	795
XIII	0.29	0.62	1.78	2.09	844
XXVIII	0.30	0.61	1.76	2.07	882
XXXII	0.30	0.60	1.76	2.04	908
XXXIV	0.31	0.65	1.87	2.14	993
XXXVII	0.30	0.66	1.87	2.14	1019
XXXVIII	0.31	0.65	1.87	2.07	1047

TABLE XXVI.—*Average negative scores for each exercise of Test D.*

Schools	Exercises			Number of Pupils
	I	II	III	
V	0.68	1.16	1.02	88
XII	0.77	1.13	1.22	132
XXIII	1.05	1.28	1.54	207
XXIV	0.84	1.09	1.42	275
XXV	1.01	1.14	1.50	350
XXVI	0.80	1.08	1.49	678
XXVII	0.93	1.00	1.50	827
XXIX	0.91	1.00	1.45	900
XLVI	0.81	0.91	1.31	1111

TABLE XXVII.—*Average negative scores for each exercise of Test E.*

Schools	Exercises				Number of Pupils
	I	II	III	IV	
I	0.34	0.33	0.72	1.22	114
II	0.30	0.28	0.63	1.05	191
III	0.32	0.24	0.68	1.07	733
IV	0.33	0.25	0.68	1.06	773
V	0.33	0.27	0.78	0.98	865
VI	0.35	0.28	0.70	0.99	962
VII	0.35	0.27	0.71	0.96	992
LV	0.36	0.28	0.71	0.97	1036

Standards of Achievements.—Tables XXVIII to XXXII give the number of pupils receiving each positive score for each test. The median scores for each school and for all pupils tested are also given. Similar data for the negative scores are given in Tables XXXIII to XXXVII.

TABLE XXVIII.—*Number of pupils receiving each positive grade for Test A.*

Score	XXIII	XXV	XXXIII	XXXV	XXXVI	XXXIX	XL	XLI	XLII	XLIII	XLIV	XLV	L	LI	LII	LIII	LIV	LVI	LIX	LX	LXI	LXIII	Totals
15	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	0
16	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1
17	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	1
18	....	1	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	2
19	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	0
20	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	0
21	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1	....	....	....	....	1
22	....	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	1	....	....	....	....	2
23	....	....	1	1	....	....	....	....	....	....	....	....	....	....	....	....	....	1	....	....	....	....	4
24	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	0
25	1	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	1	....	....	....	....	3
26	....	1	....	....	....	....	....	....	....	1	....	....	....	....	1	....	....	....	....	....	....	....	3
27	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	1	....	....	....	....	2
28	....	2	....	....	....	....	....	1	....	....	....	....	....	....	1	....	....	....	....	....	....	....	4
29	....	1	....	....	1	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	1	....	4
30	....	....	....	1	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	1	....	....	3
31	....	....	....	....	....	....	....	....	....	....	....	....	....	1	....	....	....	1	....	....	1	2	5
32	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1	....	....	1	2	5
33	....	....	....	....	....	....	....	2	....	....	....	....	....	....	....	....	....	1	....	....	....	1	4
34	1	....	....	....	....	....	....	....	1	....	....	....	....	....	1	....	....	....	....	....	1	1	5
35	1	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	2	....	1	....	1	6
36	1	....	....	....	....	....	....	....	....	1	....	....	....	....	....	1	....	2	....	....	1	1	7
37	2	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1	....	....	....	1	4
38	....	1	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	2
39	2	1	....	....	....	1	....	....	....	....	....	....	....	....	....	1	....	1	....	1	....	1	8

TABLE XXVIII.—*Continued.*

Score	XXIII	XXV	XXXIII	XXXV	XXXVI	XXXIX	XL	XLI	XLII	XLIII	XLIV	XLV	L	LI	LII	LIII	LIV	LVI	LIX	LX	LXI	LXIII	Totals
40	1	2	....	....	....	....	....	....	....	....	....	....	....	....	1	2	....	1	....	....	....	....	7
41	3	1	2	....	....	....	....	1	....	1	....	....	....	....	....	1	....	3	....	....	....	1	13
42	1	1	....	....	1	....	1	....	....	1	....	....	2	....	1	1	....	....	....	....	3	....	12
43	1	1	1	....	....	1	1	1	....	1	....	....	....	....	....	1	....	....	....	....	1	4	13
44	3	1	1	....	1	....	1	1	....	....	1	....	1	1	1	1	....	3	....	....	....	1	17
45	....	1	1	1	....	1	1	1	....	....	....	....	....	....	2	1	....	2	....	....	2	3	16
46	3	1	....	....	2	1	1	3	1	....	....	....	1	1	2	2	....	1	1	....	....	....	20
47	....	2	....	....	2	1	....	1	....	....	....	....	....	....	....	4	....	....	2	....	....	....	12
48	....	3	....	2	....	1	3	1	....	....	2	....	....	....	1	1	....	2	1	....	....	2	19
49	2	....	....	....	2	1	2	....	....	1	....	....	....	....	....	....	....	....	1	....	....	2	11
50	2	2	....	....	....	1	1	....	....	1	....	1	....	....	....	2	....	2	....	....	....	2	14
51	1	1	....	1	....	....	1	2	1	....	1	1	1	....	1	2	....	2	....	....	2	2	19
52	4	....	....	....	1	1	....	....	....	1	....	....	1	1	3	3	....	2	....	....	1	3	21
53	2	4	....	....	....	....	2	1	1	1	....	1	....	....	1	3	1	....	....	....	1	1	19
54	3	2	2	1	....	1	3	2	....	1	1	1	....	1	1	1	....	1	....	....	....	3	24
55	2	....	2	....	....	....	1	....	....	1	....	....	....	....	1	2	1	2	....	....	....	2	14
56	2	1	....	....	1	....	....	1	....	....	....	....	....	1	1	1	....	1	1	2	1	....	13
57	3	3	....	1	2	....	....	1	1	1	2	....	....	....	1	....	1	1	1	....	3	6	27
58	3	2	....	1	1	....	2	1	3	1	....	2	....	1	....	3	....	....	....	1	....	4	25
59	1	....	1	....	....	....	1	1	3	2	....	2	....	....	....	2	....	....	....	1	1	4	19
60	3	2	2	1	1	....	1	2	1	1	1	1	1	....	2	3	2	3	....	....	2	2	31
61	4	1	....	....	....	....	1	....	....	....	1	1	....	1	2	2	1	3	....	1	....	2	20
62	3	1	....	....	....	....	....	2	2	1	....	....	....	....	1	4	....	1	....	....	....	4	19
63	1	1	....	....	....	....	1	....	....	....	1	1	....	....	3	1	2	....	....	1	....	1	13
64	5	1	....	1	1	1	....	1	2	1	1	1	....	....	....	4	....	3	1	1	1	1	26

TABLE XXVIII.—*Continued.*

Score	XXIII	XXV	XXXIII	XXXV	XXXVI	XXXIX	XL	XLI	XLII	XLIII	XLIV	XLV	L	LI	LII	LIII	LIV	LVI	LIX	LX	LXI	LXIII	Totals
65	2	3	1	....	3	2	....	1	1	2	2	1	....	....	2	....	....	1	1	2	....	4	28
66	4	2	....	1	....	1	1	1	....	3	1	1	....	....	....	....	1	....	....	....	1	....	17
67	4	2	....	....	....	....	2	1	1	....	3	1	1	1	1	1	2	2	3	2	1	1	29
68	3	1	....	2	....	1	2	1	2	2	1	....	1	....	1	1	....	1	2	2	....	....	23
69	3	3	1	2	1	....	1	....	....	2	....	5	....	1	2	2	....	....	....	1	....	3	27
70	1	....	....	....	1	....	....	2	1	....	....	3	1	....	....	2	1	....	....	....	....	....	12
71	....	....	....	....	....	....	1	2	2	2	1	1	....	....	1	3	....	3	....	1	....	1	18
72	3	2	....	1	....	....	3	2	5	....	1	3	2	3	....	3	1	2	3	1	....	1	36
73	2	1	....	....	1	....	....	1	1	....	1	....	....	1	....	1	2	....	....	1	....	....	12
74	1	5	....	1	1	....	1	3	2	5	1	12	....	....	....	2	4	5	2	7	....	7	59
75	2	....	....	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	3	1	....	1	8
76	1	2	....	....	....	....	1	....	....	....	1	....	....	....	1	1	....	....	....	....	....	2	9
77	....	3	....	1	....	....	....	....	1	1	1	....	....	....	....	....	....	....	....	....	....	....	7
78	....	1	....	2	....	....	....	....	2	2	3	....	....	....	1	1	1	1	3	....	....	....	17
79	....	....	....	....	....	....	....	1	....	....	1	....	....	....	1	....	....	....	....	....	....	....	3
80	....	....	....	1	....	....	2	....	1	....	1	1	....	....	....	....	....	....	....	....	....	1	7
81	....	....	....	....	1	....	....	....	2	....	2	....	2	....	1	....	....	....	1	....	....	....	9
82	....	....	....	....	....	....	....	....	2	....	3	....	....	....	1	1	....	....	1	....	....	1	9
83	....	1	....	1	....	....	1	2	4	1	2	1	....	....	....	....	....	....	....	1	....	2	16
84	....	....	....	....	....	....	....	....	....	2	1	....	....	....	....	....	....	....	....	....	....	....	3
85	....	....	....	....	....	....	....	....	....	....	2	....	....	....	....	....	....	....	....	....	....	....	2
86	....	....	....	....	....	....	....	....	1	....	1	1	....	....	....	....	....	....	1	....	....	....	4
87	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	1
88	....	....	....	....	....	....	1	1	....	2	....	....	....	....	1	....	....	....	....	....	....	1	6
89	....	....	....	....	....	....	2	....	....	3	....	1	....	....	2	1	....	....	....	....	....	....	9

TABLE XXVIII.—*Continued.*

Score	XXXIII	XXV	XXXIII	XXXV	XXXVI	XXXIX	XL	XLI	XLII	XLIII	XLIV	XLV	L	LI	LII	LIII	LIV	LVI	LIX	LX	LXI	LXIII	Totals
90	....	....	....	....	....	1	....	1	....	....	1	....	....	....	2	....	....	....	....	....	....	....	5
91	1	....	....	....	....	....	....	....	....	3	4	4	....	....	1	1	2	1	....	....	....	....	17
92	....	....	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	1
93	....	....	....	....	....	....	....	1	....	1	1	1	....	....	....	....	....	....	....	....	....	....	4
94	....	....	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	1
95	....	....	....	....	....	....	....	1	1	....	2	1	....	....	....	....	....	....	1	....	....	....	6
96	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	1
97	....	....	....	....	....	....	1	....	....	....	....	....	....	....	1	....	....	....	....	....	....	1	3
98	....	....	....	....	....	....	....	....	1	1	....	....	....	....	....	....	....	....	....	....	....	....	2
99	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1
100	....	....	....	....	....	....	1	1	3	1	2	1	....	....	1	1	....	....	....	....	....	1	12
Totals.....	88	70	15	23	26	17	47	48	51	53	52	51	14	14	48	71	22	63	29	29	25	88	944
Median.....	59.0	57.7	54.8	66.5	57.0	50.5	60.5	62.5	72.5	68.8	78.7	72.5	64.0	60.0	62.0	59.8	69.0	52.8	72.2	68.3	57.8	58.3	62.5
25 percentile ...	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	51.3
75 percentile ...	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	72.9
Quartile.....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	10.8

TABLE XXIX.—*Number of pupils receiving each positive grade for Test B.*

Score	XIV	XV	XVI	XVII	XVIII	XIX	XX	XXI	XXV	XXX	XXXI	XLVII	XLVIII	XLIX	LVII	LVIII	Totals
0	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
2	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
3	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
4	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
5	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
6	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
7	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
8	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	1
9	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
10	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	1
11	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	1
12	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	1
13	.....	.....	.....	.....	.....	.....	.....	.....	.....	2	1	.....	.....	.....	.....	1	4
14	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
15	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
16	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
17	1	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	2
18	.....	.....	.....	.....	.....	.....	.....	2	.....	.....	1	.....	.....	.....	.....	.....	3
19	.....	.....	.....	1	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	2
20	2	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	3
21	.....	.....	.....	1	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	2
22	1	.....	1	.....	.....	.....	.....	.....	1	1	.....	.....	.....	.....	.....	.....	4
23	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	1
24	.....	.....	.....	1	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	1	.....	3

TABLE XXIX.—*Continued.*

Score	XIV	XV	XVI	XVII	XVIII	XIX	XX	XXI	XXV	XXX	XXXI	XLVII	XLVIII	XLIX	LVII	LVIII	Totals
25	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	1	.....	1	3
26	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	2	.....	.....	.....	.....	.....	3
27	.....	.....	.....	.....	.....	.....	.....	.....	1	1	.....	.....	.....	.....	.....	.....	2
28	.....	.....	.....	1	.....	.....	.....	1	2	.....	1	.....	.....	.....	.....	2	7
29	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	1
30	.....	.....	.....	.....	1	.....	.....	2	1	3	1	.....	.....	.....	.....	1	9
31	1	.....	.....	1	1	.....	.....	2	.....	2	.....	1	.....	.....	.....	1	9
32	1	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	2
33	2	.....	1	.....	.....	.....	.....	1	.....	1	.....	.....	.....	.....	.....	.....	5
34	.....	.....	2	.....	1	.....	1	1	.....	.....	.....	.....	.....	1	.....	.....	6
35	.....	.....	.....	.....	.....	.....	.....	1	3	.....	.....	.....	.....	2	1	.....	7
36	1	.....	1	.....	.....	.....	.....	.....	2	2	.....	.....	.....	1	.....	1	8
37	.....	.....	.....	1	1	.....	.....	1	.....	1	.....	1	.....	1	2	.....	8
38	1	.....	.....	.....	.....	.....	.....	1	1	2	.....	.....	.....	1	.....	1	7
39	.....	.....	.....	.....	1	.....	2	5	.....	2	.....	.....	.....	.....	1	2	13
40	.....	.....	1	2	.....	.....	1	2	.....	1	2	.....	.....	.....	.....	.....	9
41	.. ..	1	1	.....	.....	1	1	2	.....	1	.....	.....	.....	1	1	1	10
42	.....	.....	.....	.....	.....	.....	1	3	1	2	1	.....	.....	2	.....	.....	10
43	.....	1	.....	1	.....	.....	.....	3	3	3	2	1	.....	.....	2	.....	16
44	.....	1	.....	.....	.....	.....	3	1	1	2	.....	.....	.....	1	1	2	12
45	.....	1	.....	.....	.....	1	3	.....	1	.....	4	.....	.....	1	.....	1	12
46	.....	.....	.....	.....	1	.....	.....	2	.....	1	1	.....	1	2	.....	1	9
47	.....	.....	1	2	1	1	1	2	3	3	.....	2	1	.....	.....	.....	17
48	.....	.....	.....	1	1	.....	.....	3	.....	1	3	.....	.....	.....	1	1	11
49	.....	.....	.....	.....	1	2	3	.....	1	2	.....	.....	.....	.....	.....	.....	9

TABLE XXIX.—*Continued.*

Score	XIV	XV	XVI	XVII	XVIII	XIX	XX	XXI	XXV	XXX	XXXI	XLVII	XLVIII	XLIX	LVII	LVIII	Totals
50	.....	.....	.....	.....	.....	.....	3	.....	2	2	1	3	.....	2	1	1	15
51	1	.....	.....	1	.....	.....	.....	2	1	.....	.....	.....	.....	1	1	.....	7
52	.....	.....	.....	2	.....	.....	1	.....	1	.....	1	1	.....	2	1	.....	9
53	.....	.....	.....	1	1	1	3	.....	3	2	2	1	1	.....	1	.....	16
54	.....	.....	.....	1	1	.....	1	.....	.....	1	.....	.....	.....	1	.....	2	7
55	1	1	1	.....	3	.....	2	1	1	.....	1	.....	.....	.....	2	1	14
56	.....	1	.....	1	.....	1	4	2	.....	.....	2	1	1	1	1	.....	15
57	.....	.....	.....	.....	2	.....	2	1	.....	.....	.....	1	.....	2	.....	1	9
58	.....	.....	1	1	.....	1	2	1	.....	1	3	1	.....	2	.....	.....	13
59	.....	.....	1	.....	1	.....	1	.....	.....	.....	.....	1	3	1	.....	1	9
60	.....	.....	.....	.....	.....	1	4	1	2	.....	3	.....	1	1	2	2	17
61	.....	.....	1	1	2	.....	3	2	.....	.....	.....	.....	2	3	3	.....	17
62	.....	.....	1	.....	3	1	3	.....	1	1	1	.....	.....	2	.....	1	14
63	1	.....	.....	2	3	1	7	.....	.....	.....	1	.....	1	1	1	2	20
64	.....	.....	1	.....	2	2	3	1	1	.....	.....	.....	.....	.....	.....	1	11
65	.....	.....	1	.....	4	2	1	.....	1	1	1	.....	5	1	.....	3	20
66	.....	.....	1	.....	3	1	4	1	1	.....	1	2	1	4	.....	.....	19
67	.....	.....	1	1	2	1	7	3	.....	.....	1	2	.....	2	.....	.....	20
68	1	.....	2	.....	4	1	4	.....	3	.....	.....	.....	2	2	.....	2	21
69	.....	.....	3	.....	2	.....	2	1	.....	1	1	.....	1	.....	.....	1	12
70	.....	1	1	.....	4	2	6	1	2	3	4	1	2	2	.....	3	32
71	1	.....	.....	.....	6	.....	5	3	.....	.....	1	.....	2	7	.....	1	26
72	.....	.....	1	.....	4	2	5	.....	2	.....	.....	.....	.....	2	1	1	18
73	1	.....	.....	.....	5	1	4	.....	.....	.....	1	1	.....	.....	1	.....	14
74	.....	1	1	.....	7	2	1	3	.....	1	1	.....	4	3	.....	4	28
75	.....	1	2	.....	4	4	3	.....	3	.....	2	1	1	4	2	.....	27
76	.....	.....	1	.....	6	2	4	.....	.....	1	1	.....	.....	1	.....	.....	16
77	.....	1	1	2	1	.....	2	.....	1	.....	.....	2	1	1	1	.....	13
78	.....	1	2	.....	2	1	3	.....	.....	.....	1	3	.....	2	1	.....	16
79	.....	.....	1	1	.....	3	6	1	1	.....	1	1	.....	1	1	.....	17

TABLE XXIX.—*Continued.*

Score	XIV	XV	XVI	XVII	XVIII	XIX	XX	XXI	XXV	XXX	XXXI	XLVII	XLVIII	XLIX	LVII	LVIII	Totals
80	1	1	.....	.....	4	4	4	.....	1	.....	.....	.....	3	2	1	.....	21
81	.....	.....	.....	.....	4	1	6	.....	2	.....	.....	.....	.....	2	1	.....	16
82	.....	.....	1	1	1	5	6	2	1	.....	.....	.....	.....	2	.....	.....	19
83	.....	.....	3	.....	4	.....	3	.....	2	1	.....	.....	.....	2	.....	.....	15
84	.....	.....	2	.....	.....	2	4	1	2	.....	.....	.....	.....	1	.....	.....	12
85	.....	.....	.....	.....	3	3	1	.....	.....	.....	.....	2	1	.....	.....	.....	10
86	.....	.....	.....	.....	5	2	5	.....	1	.....	.....	2	.....	.....	.....	.....	15
87	.....	.....	.....	1	8	3	6	2	1	.....	1	1	1	.....	1	.....	25
88	1	.....	1	2	2	5	2	.....	.....	.....	.....	.....	2	2	.....	1	18
89	.....	.....	.....	2	3	.....	2	.....	2	.....	1	.....	.....	1	1	.....	12
90	.....	.....	2	.....	7	2	8	1	1	.....	.....	.....	.....	3	2	1	27
91	.....	.....	1	1	3	1	.....	.....	2	.....	.....	1	.....	.....	.....	1	10
92	.....	.....	1	.....	4	2	3	.....	.....	.....	.....	.....	4	.....	.....	1	15
93	1	3	.....	1	1	3	2	.....	.....	.....	.....	1	2	1	.....	3	18
94	.....	.....	.....	.....	3	3	4	1	.....	.....	.....	.....	1	2	.....	1	15
95	.....	.....	.....	.....	2	.....	4	.....	.....	.....	.....	1	1	2	1	.....	11
96	.....	.....	1	.....	3	4	2	.....	.....	1	.....	.....	1	1	.....	.....	13
97	.....	.....	2	.....	3	2	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	8
98	.....	.....	1	.....	1	.....	1	1	.....	.....	.....	.....	1	.....	.....	1	6
99	.....	.....	2	.....	3	.....	1	.....	.....	.....	.....	1	.....	.....	1	.....	8
100	.....	.....	1	.....	1	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	3
Totals.....	19	15	51	36	146	77	177	72	66	54	53	36	47	86	38	52	1025
Medians.....	38.5	74.5	74.5	54.0	76.2	80.9	72.7	47.5	61.5	43.3	56.8	69.0	71.8	70.5	61.0	63.5	69.3
25 percentile .	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	51.8
75 percentile .	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	82.2
Quartile.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	15.2

TABLE XXX.—*Number of pupils receiving each positive score for Test C.*

Score	VII	VIII	IX	X	XI	XII	XIII	XXVIII	XXXII	XXXIV	XXXVII	XXXVIII	Totals
1	....	....	....	....	....	....	....	1	....	....	....	....	1
2	....	....	....	....	....	....	....	....	....	....	....	....	0
3	....	....	....	....	....	....	....	2	....	....	....	....	2
4	....	1	1	....	....	....	....	....	....	1	....	....	3
5	....	....	....	....	....	....	....	....	....	....	....	....	0
6	....	1	....	....	....	....	....	....	....	....	....	....	1
7	....	1	....	....	....	....	....	....	....	....	....	....	1
8	....	....	....	....	....	....	....	1	....	1	....	....	2
9	....	1	....	....	....	1	....	....	....	....	....	....	2
10	....	1	1	....	....	1	....	3	....	....	....	....	6
11	....	1	....	....	....	1	....	....	....	....	....	....	2
12	....	....	1	....	....	....	....	1	....	1	....	....	3
13	....	3	....	....	....	1	....	1	....	1	....	1	7
14	....	1	2	....	....	1	1	1	1	1	....	....	8
15	....	....	....	....	....	1	....	1	....	2	....	....	4
16	....	1	1	....	....	1	....	....	....	....	....	....	3
17	....	1	....	....	....	1	....	....	....	....	....	....	2
18	....	2	2	....	....	....	....	1	....	....	....	....	5
19	....	2	1	....	....	1	....	....	....	1	....	....	5
20	....	1	....	....	....	....	....	3	....	....	....	1	5
21	1	2	....	....	....	....	....	1	1	....	....	....	5
22	....	3	2	....	....	1	....	....	....	1	....	....	7
23	....	2	1	....	....	1	3	....	1	3	....	....	11
24	....	8	....	1	....	1	....	....	....	....	....	....	10
25	....	4	3	....	1	4	1	....	....	2	....	2	17
26	....	4	1	....	....	3	....	....	....	1	....	....	9
27	....	2	1	....	3	1	1	2	....	1	....	....	11
28	....	5	1	....	....	....	2	1	1	1	....	2	12
29	....	8	1	1	1	3	2	1	1	....	....	1	19
30	....	4	1	....	3	3	2	....	....	....	....	1	14
31	....	7	1	....	....	....	....	1	2	1	1	1	14
32	....	8	....	2	....	1	2	1	....	1	....	....	15
33	....	8	....	....	....	3	1	....	....	1	....	....	13
34	....	8	....	....	....	3	1	1	....	....	1	....	14
35	2	7	2	2	....	2	2	1	....	....	1	....	19
36	....	12	....	1	....	2	3	1	....	1	....	1	21
37	....	8	1	2	....	4	....	2	2	2	....	1	22
38	....	10	2	1	1	2	....	....	....	....	....	1	17
39	....	8	2	1	....	2	1	....	....	1	....	....	15
40	....	5	1	1	2	....	2	....	....	....	....	....	11
41	....	17	3	....	....	1	....	1	1	....	1	....	24
42	....	7	1	....	....	2	1	....	1	1	....	....	13
43	....	6	2	2	2	3	2	....	1	1	1	....	20
44	....	7	4	....	....	2	1	....	1	1	....	....	16
45	1	7	....	1	....	3	1	1	1	1	1	1	18

TABLE XXX.—*Continued.*

Score	VII	VIII	IX	X	XI	XII	XIII	XXVIII	XXXII	XXXIV	XXXVII	XXXVIII	Totals
46	1	9	1	2	....	4	1	1	1	1	1	....	22
47	....	13	....	....	....	....	....	1	....	2	....	1	17
48	1	7	2	....	....	1	....	2	1	2	1	1	18
49	....	12	1	1	2	....	....	1	....	....	1	1	19
50	1	16	2	3	....	4	2	....	....	2	1	1	32
51	....	11	....	....	1	1	1	1	....	1	....	1	17
52	....	14	2	1	....	1	1	1	1	2	1	2	26
53	....	7	1	....	....	2	....	....	....	1	....	....	11
54	....	13	3	....	....	2	....	....	....	3	1	2	24
55	....	13	1	2	2	....	....	....	....	3	1	....	22
56	....	13	....	....	....	1	1	....	....	3	1	....	19
57	....	6	1	....	1	....	....	1	....	2	....	....	11
58	....	12	1	....	....	....	1	....	....	1	1	....	16
59	1	7	....	1	1	1	....	....	2	1	....	....	14
60	1	3	1	1	....	5	....	....	....	....	1	1	13
61	1	8	1	....	1	1	1	....	1	3	4	....	21
62	1	8	....	....	....	1	....	....	....	1	....	....	11
63	....	11	1	....	1	....	1	1	....	3	....	....	18
64	2	14	....	2	....	....	2	....	....	1	....	1	22
65	....	11	....	....	....	3	2	....	1	1	....	....	18
66	2	9	2	....	1	....	1	....	....	1	....	....	16
67	2	13	....	....	....	....	....	....	....	....	1	....	16
68	1	11	1	1	....	....	2	....	....	3	....	1	20
69	....	8	....	....	1	....	....	....	2	4	....	1	16
70	....	7	2	....	....	....	1	....	....	1	....	....	11
71	....	10	....	....	1	....	....	....	....	2	....	1	14
72	....	9	....	....	1	1	1	....	....	3	....	....	15
73	1	4	2	....	1	....	1	....	....	....	....	....	9
74	....	8	....	1	....	....	....	....	....	....	....	....	9
75	1	2	1	....	....	....	....	....	1	1	....	....	6
76	1	4	....	....	....	....	1	....	....	1	....	....	7
77	....	10	....	....	....	....	....	....	....	1	....	....	11
78	1	2	2	....	....	....	....	....	1	....	....	....	6
79	....	3	....	1	....	....	....	....	....	....	....	....	4
80	1	7	2	....	....	....	....	....	....	....	....	....	10
81	....	6	2	....	....	....	....	....	....	....	....	....	8
82	....	5	2	....	....	....	....	....	....	1	....	....	8
83	....	5	....	....	....	....	....	....	....	....	....	....	5
84	....	....	....	....	....	....	....	....	....	....	2	....	2
85	2	3	....	....	....	....	....	....	....	1	....	....	6
86	....	2	....	....	....	....	....	....	....	1	1	....	4
87	....	2	....	....	....	....	....	....	....	1	1	....	4
88	1	2	....	....	....	1	....	....	....	....	1	....	5
89	....	4	....	....	....	....	....	....	....	....	....	....	4
90	....	....	....	....	1	....	....	....	....	1	....	....	2

TABLE XXX.—*Continued.*

Score	VII	VIII	IX	X	XI	XII	XIII	XXVIII	XXXII	XXXIV	XXXVII	XXXVIII	Totals
91	....	2	....	....	....	....	....	....	....	....	....	1	3
92	1	5	....	....	....	....	....	....	....	....	....	....	6
93	2	1	....	....	....	....	....	....	....	....	....	....	3
94	....	....	....	....	....	....	....	....	....	....	....	1	1
95	....	1	....	....	....	....	....	....	1	....	....	....	2
96	....	3	....	....	....	....	....	....	....	....	....	....	3
97	1	....	....	....	....	....	....	....	....	....	....	....	1
98	....	1	....	....	....	....	....	....	....	1	....	....	2
99	....	1	....	....	....	....	....	....	....	1	....	....	2
100	....	6	....	....	....	....	....	....	....	....	....	....	6
Totals	30	548	73	31	28	85	49	38	26	85	26	28	1047
Medians	67.0	54.1	44.4	46.3	49.5	38.8	42.5	29.0	45.0	55.2	56.0	49.0	50.6
25 percentile	....	....	....	....	....	....	....	....	....	....	....	....	36.5
75 percentile	....	....	....	....	....	....	....	....	....	....	....	....	65.2
Quartile	....	....	....	....	....	....	....	....	....	....	....	....	14.4

TABLE XXXI.—*Number of pupils receiving each positive score for Test D.*

Score	V	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXIX	XLVI	Totals
0	.....	.....	.....	.....	.....	.....	.....	1	.....	1
1	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
7	.....	.....	.....	.....	.....	.....	.....	2	.....	2
8	.....	.....	.....	.....	.....	.....	.....	2	.....	2
9	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
10	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
11	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
12	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
13	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
14	.....	.....	.....	.....	.....	1	.....	2	1	4
15	.....	.....	.....	.....	.....	.....	.....	1	.....	1
16	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
17	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
18	.....	.....	.....	.....	.....	.....	.....	1	.....	1
19	.....	.....	.....	.....	.....	.....	.....	1	.....	1
20	.....	.....	2	.....	.....	1	.....	1	1	5
21	.....	1	.....	.....	.....	.....	.....	.....	.....	1
22	.....	.....	1	1	.....	.....	.....	.....	.....	2
23	.....	.....	.....	.....	1	.....	.....	1	.....	2
24	.....	.....	.....	.....	.....	.....	.....	1	.....	1
25	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
26	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
27	.....	2	4	1	1	2	.....	3	3	16
28	.....	.....	.....	.....	.....	1	.....	.....	.....	1
29	.....	.....	.....	.....	2	.....	.....	1	.....	3
30	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
31	1	.....	.....	.....	.....	.....	.....	.....	.....	1
32	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
33	.....	.....	1	1	2	3	.....	1	1	9
34	.....	.....	2	2	1	5	1	.....	3	14
35	1	.....	.....	.....	1	.....	.....	.....	.....	2
36	.....	.....	.....	.....	4	.....	.....	.....	1	5
37	1	.....	.....	.....	2	1	.....	.....	.....	4
38	.....	1	.....	.....	1	1	1	.....	.....	4
39	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
40	3	2	2	1	.....	15	2	1	5	31
41	.....	.....	3	1	.....	9	3	4	2	22
42	1	.....	.....	.....	1	.....	1	1	.....	4
43	1	.....	.....	.....	.....	.....	.....	.....	.....	1
44	1	.....	.....	.....	3	1	.....	.....	.....	5
45	.....	.....	.....	.....	.....	1	1	.....	.....	2
46	1	2	1	.....	1	.....	.....	.....	2	7
47	.....	2	10	3	2	7	3	5	7	39
48	.....	.....	1	.....	.....	.....	1	3	1	6
49	1	.....	.....	.....	.....	2	1	.....	.....	4
50	1	1	.....	.....	1	1	1	1	2	8
51	.....	.....	1	.....	.....	1	2	.....	1	5
52	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
53	3	1	2	.....	1	8	3	.....	5	23
54	3	2	3	4	.....	10	.....	5	10	37

TABLE XXXI.—*Continued.*

Score	V	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXIX	XLVI	Totals
55	.....	.....	.....	.....	.....	.....	5	.....	.....	5
56	.....	.....	.....	.....	1	4	.....	3	.....	8
57	2	.....	.....	.....	2	1	.....	1	2	8
58	.....	.....	.....	.....	2	.....	1	.....	.....	3
59	1	.....	.....	.....	3	.....	1	.....	1	6
60	.....	1	.....	.....	1	11	3	1	5	22
61	4	.....	3	9	7	17	12	1	9	62
62	1	1	.....	1	1	6	2	2	.....	14
63	1	1	.....	.....	1	.....	1	3	.....	7
64	1	2	.....	2	.....	6	.....	.....	3	14
65	.....	.....	.....	.....	.....	2	1	.....	1	4
66	1	.....	4	1	1	4	1	.....	6	18
67	1	.....	3	4	6	15	3	1	12	45
68	2	.....	.....	4	2	4	3	2	2	19
69	1	1	1	2	.....	2	.....	.....	1	8
70	3	1	.....	.....	2	.....	1	.....	2	9
71	2	2	3	2	1	1	3	.....	.....	14
72	1	.....	.....	.....	1	1	.....	1	.....	4
73	1	.....	2	3	.....	19	3	.....	6	34
74	4	3	2	2	5	22	7	6	16	67
75	4	.....	.....	.....	.....	1	.....	.....	.....	5
76	.....	1	3	.....	.....	3	1	.....	.....	8
77	.....	1	2	1	.....	2	1	1	1	9
78	.....	.....	1	.....	1	5	1	.....	4	12
79	1	.....	.....	.....	.....	2	.....	1	2	6
80	5	2	6	2	4	20	9	.....	4	52
81	1	3	.....	2	.....	3	5	2	6	22
82	.....	.....	.....	.....	.....	1	.....	.....	1	2
83	4	1	.....	5	.....	5	5	1	3	24
84	3	.....	1	1	1	4	2	3	3	18
85	.....	.....	.....	.....	1	1	1	.....	.....	3
86	.....	1	1	3	.....	10	1	.....	9	25
87	11	.....	2	1	3	31	12	1	10	71
88	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
89	1	.....	1	.....	.....	7	4	.....	.....	13
90	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
91	4	.....	1	3	2	6	5	1	8	30
92	3	.....	.....	1	1	.....	.....	.....	2	7
93	1	3	4	.....	.....	12	9	.....	12	41
94	1	.....	.....	.....	.....	.....	.....	1	.....	2
95	.....	.....	.....	.....	.....	.....	.....	.....	3	3
96	.....	2	.....	.....	.....	5	2	.....	1	10
97	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
98	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
99	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
100	5	4	2	5	2	25	24	3	31	101
Totals.....	88	44	75	68	75	328	149	73	211	1111
Medians	75.0	71.5	66.4	68.8	61.6	74.0	80.5	54.7	74.7	73.3
25 percentile.	.....	.....	.....	.....	.....	.....	.....	.....	.....	55.4
75 percentile.	.....	.....	.....	.....	.....	.....	.....	.....	.....	87.0
Quartile.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	15.8

TABLE XXXII.—*Number of pupils receiving each positive score for Test E.*

Score	I	II	III	IV	V	VI	VII	LV	Totals
0	.....	.....	.....	.....	.....	1	.....	.....	1
1	.....	.....	.....	.....	.....	.....	.....	.....	0
11	.....	.....	.....	.....	.....	.....	.....	.....	0
12	1	1	8	1	3	5	.....	2	21
13	.....	.....	.....	.....	.....	.....	.....	.....	0
15	.....	.....	.....	.....	.....	.....	.....	.....	0
16	1	1	6	.....	.....	2	.....	.....	10
17	.....	.....	.....	.....	.....	.....	.....	.....	0
27	.....	.....	.....	.....	.....	.....	.....	.....	0
28	26	20	166	13	6	56	8	8	303
29	.....	.....	.....	.....	.....	.....	.....	.....	0
44	.....	.....	.....	.....	.....	.....	.....	.....	0
45	.....	.....	12	.....	.....	2	.....	2	16
46	.....	.....	.....	.....	.....	.....	.....	.....	0
48	.....	.....	.....	.....	.....	.....	.....	.....	0
49	1	.....	5	.....	.....	1	.....	.....	7
50	.....	.....	.....	.....	.....	.....	.....	.....	0
51	.....	.....	1	.....	.....	1	.....	2	4
52	.....	.....	.....	.....	.....	.....	.....	.....	0
54	.....	.....	.....	.....	.....	.....	.....	.....	0
55	.....	1	.....	1	.....	2	.....	.....	4
56	.....	.....	.....	.....	.....	.....	.....	.....	0
60	.....	.....	.....	.....	.....	.....	.....	.....	0
61	49	16	198	12	5	25	5	4	314
62	.....	.....	.....	.....	.....	.....	.....	.....	0
66	.....	.....	.....	.....	.....	.....	.....	.....	0
67	9	14	52	3	25	1	12	13	129
68	.....	.....	.....	.....	.....	.....	.....	.....	0
71	.....	.....	.....	.....	.....	.....	.....	.....	0
72	.....	.....	.....	.....	1	.....	.....	.....	1
73	.....	.....	.....	.....	.....	.....	.....	.....	0
83	.....	.....	.....	.....	.....	.....	.....	.....	0
84	.....	.....	4	.....	4	.....	.....	1	9
85	.....	.....	.....	.....	.....	.....	.....	.....	0
87	.....	.....	.....	.....	.....	.....	.....	.....	0
88	.....	.....	2	.....	.....	.....	.....	.....	2
89	.....	.....	.....	.....	.....	.....	.....	.....	0
99	.....	.....	.....	.....	.....	.....	.....	.....	0
100	27	24	88	10	48	1	5	12	215
Totals.....	114	77	542	40	92	97	30	44	1036
Medians.....	61.6	62.0	61.4	61.4	99.0	28.7	67.2	67.3	61.5
25 percentile..	.....	.....	.....	.....	.....	.....	.....	.....	28.7
75 percentile..	.....	.....	.....	.....	.....	.....	.....	.....	67.7
Quartile.....	.....	.....	.....	.....	.....	.....	.....	.....	19.5

TABLE XXXIII.—*Number of pupils receiving each negative score for Test A.*

Score	XXIII	XXV	XXXIII	XXXV	XXXVI	XXXIX	XL	XLI	XLII	XLIII	XLIV	XLV	L	LI	LII	LIII	LIV	LVI	LIX	LX	LXI	LXIII	Totals
0	1	2	....	....	1	....	4	5	6	6	3	2	....	....	2	....	....	1	1	....	....	3	37
1	2	3	....	....	....	2	....	4	1	1	5	1	....	1	3	1	4	6	....	....	4	....	38
2	4	3	1	1	1	2	2	3	7	7	4	1	3	....	6	....	4	3	3	....	2	8	65
3	2	3	....	2	5	3	5	4	6	3	4	3	1	2	9	3	....	8	3	2	4	10	82
4	7	9	1	3	2	2	4	4	3	5	4	2	....	2	2	5	1	9	1	1	2	3	72
5	8	6	3	2	1	1	3	6	4	2	2	4	2	1	3	2	2	6	2	2	4	11	77
6	6	10	4	3	6	2	2	4	7	....	1	5	....	3	9	4	....	4	2	1	4	16	93
7	10	5	1	5	2	1	4	6	1	6	4	6	1	2	4	7	3	4	4	2	....	10	88
8	8	7	1	2	1	2	5	4	7	4	2	3	1	....	3	3	5	5	4	1	....	9	77
9	4	4	3	1	1	1	1	2	3	2	1	4	2	1	....	3	2	2	1	3	3	5	49
10	7	9	....	....	3	1	3	1	1	3	2	....	....	2	3	....	1	1	....	1	2	4	44
11	11	3	....	....	1	....	3	2	3	2	5	5	1	....	1	10	....	9	2	4	....	2	64
12	4	1	....	....	....	....	4	....	....	4	4	2	....	....	1	8	....	2	2	3	....	3	38
13	....	....	....	1	....	....	4	2	1	5	5	4	3	....	1	5	....	3	2	4	....	2	42
14	6	1	....	2	....	....	1	....	....	1	3	4	....	....	....	7	....	....	2	....	....	1	28
15	3	4	....	....	....	....	1	1	....	2	2	3	....	....	....	3	....	....	....	2	....	....	21
16	1	....	....	1	....	....	....	....	....	....	1	1	....	....	1	3	....	....	....	2	....	....	10
17	2	....	....	....	....	....	....	....	1	....	....	....	....	....	....	1	....	....	....	....	....	1	5
18	1	....	....	....	1	....	....	....	....	....	....	1	....	....	....	2	....	....	....	....	....	....	5
19	1	....	....	....	1	....	1	....	....	....	....	....	....	....	....	3	....	....	....	1	....	....	7
20	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	0
21	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	0
22	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	1
23	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1
Totals	88	70	15	23	26	17	47	48	51	53	52	51	14	14	48	71	22	63	29	29	25	88	944
Medians	8.5	6.9	6.6	7.1	6.5	4.8	7.9	5.7	5.6	7.4	7.8	8.5	8.0	6.3	5.7	11.8	6.5	5.8	7.6	11.4	5.1	6.6	7.1
25 percentile	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	4.2
75 percentile	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	10.7
Quartile	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	3.3

TABLE XXXIV.—*Number of pupils receiving each negative score for Test B.*

Score	XIV	XV	XVI	XVII	XVIII	XIX	XX	XXI	XXV	XXX	XXXI	XLVII	XLVIII	XLIX	LVII	LVIII	Totals
0	4	5	13	15	32	26	38	21	12	7	7	7	15	18	8	12	240
1	2	1	2	3	10	9	16	5	5	6	6	8	4	9	4	5	95
2	1	2	8	4	11	6	18	7	5	5	5	4	2	7	4	5	94
3	3	3	13	3	37	15	32	8	13	8	4	6	6	14	4	5	174
4	2	1	2	3	15	10	16	6	7	4	10	.....	7	12	2	5	102
5	.....	2	2	.....	9	1	15	6	2	7	3	3	2	11	6	5	74
6	.....	1	5	3	18	6	16	6	9	6	6	2	5	4	2	4	93
7	3	.....	.....	2	2	1	5	6	2	3	4	5	2	4	4	3	46
8	.....	.....	2	1	5	.....	10	2	3	4	3	1	2	3	1	2	39
9	1	.....	2	.....	5	.....	5	4	2	1	.....	.....	.....	1	.....	1	22
10	1	.....	1	.....	1	1	3	1	.....	1	.....	.....	1	1	.....	3	14
11	.....	.....	.....	1	.....	2	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	3
12	1	.....	1	1	.....	.....	2	.....	1	1	1	.....	1	2	2	1	14
13	.....	.....	.....	.....	1	.....	.....	.....	2	1	2	.....	.....	.....	1	.....	7
14	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	1
15	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	2	.....	.....	.....	.....	1	4
16	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
17	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
18	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
19	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
20	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
21	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
22	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
23	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
24	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	1
Totals.....	19	15	51	36	146	77	177	72	66	54	53	36	47	86	38	52	1025
Medians.....	3.8	2.8	3.2	2.0	3.5	2.6	3.5	3.4	3.8	4.3	4.5	2.8	3.4	3.6	3.8	3.8	3.5
25 percentile .	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.2
75 percentile .	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.9
Quartile.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	2.4

TABLE XXXV.—*Number of pupils receiving each negative score for Test C.*

Score	VII	VIII	IX	X	XI	XII	XIII	XXVIII	XXXII	XXXIV	XXXVII	XXXVIII	Totals
0	3	85	9	7	4	12	6	12	1	5	3	5	152
1	5	57	7	6	3	14	6	3	4	5	2	5	117
2	5	68	10	6	3	8	9	4	2	5	5	1	126
3	2	67	6	3	4	7	6	5	6	5	6	5	122
4	5	62	4	3	3	9	7	4	5	10	2	4	118
5	3	46	2	3	3	2	1	....	4	3	2	....	69
6	....	29	9	1	2	4	7	1	....	6	3	2	64
7	....	30	4	1	1	6	1	4	2	13	1	4	67
8	1	15	4	1	2	3	....	....	....	2	....	....	28
9	3	13	3	....	....	3	....	1	1	3	....	....	27
10	....	12	2	....	2	4	1	2	....	4	....	....	27
11	1	12	3	....	....	1	1	....	1	1	....	1	21
12	1	14	3	....	....	4	2	....	....	4	....	....	28
13	....	5	3	....	1	4	1	1	....	4	1	....	20
14	....	5	....	....	....	2	....	1	....	1	....	....	9
15	....	7	4	....	....	....	....	....	....	....	....	....	11
16	1	5	....	....	....	....	1	....	....	1	....	1	9
17	....	2	....	....	....	....	....	....	....	2	....	....	4
18	....	4	....	....	....	....	....	....	....	2	....	....	6
19	....	2	....	....	....	....	....	....	....	1	....	....	3
20	....	....	....	....	....	....	....	....	....	2	....	....	2
21	....	1	....	....	....	....	....	....	....	1	....	....	2
22	....	....	....	....	....	....	....	....	....	....	....	....	0
23	....	3	....	....	....	....	....	....	....	1	....	....	4
24	....	1	....	....	....	1	....	....	....	1	....	....	3
25	....	....	....	....	....	....	....	....	....	3	....	....	3
26	....	....	....	....	....	....	....	....	....	....	....	....	0
27	....	....	....	....	....	....	....	....	....	....	....	....	0
28	....	....	....	....	....	....	....	....	....	....	....	....	0
29	....	....	....	....	....	....	....	....	....	....	....	....	0
30	....	....	....	....	....	....	....	....	....	....	....	....	0
31	....	....	....	....	....	....	....	....	....	....	....	....	0
32	....	....	....	....	....	....	....	....	....	....	....	....	0
33	....	1	....	....	....	....	....	....	....	....	....	....	1
34	....	....	....	....	....	1	....	....	....	....	....	....	1
35	....	1	....	....	....	....	....	....	....	....	1	....	2
36	....	1	....	....	....	....	....	....	....	....	....	....	1
Totals.....	30	548	73	31	28	85	49	38	26	85	26	28	1047
Medians.....	4.0	4.0	5.3	2.4	4.0	4.2	3.6	3.0	4.0	7.3	3.5	3.6	4.1
25 percentile ..	....	....	....	....	....	....	....	....	....	....	....	....	1.9
75 percentile ..	....	....	....	....	....	....	....	....	....	....	....	....	7.3
Quartile.....	....	....	....	....	....	....	....	....	....	....	....	....	2.7

TABLE XXXVI.—*Number of pupils receiving each negative grade for Test D.*

Score	V	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXIX	XLVI	Totals
0	18	13	11	26	8	92	44	25	62	299
1	17	3	15	14	13	57	20	15	28	182
2	17	7	9	13	8	42	13	3	22	134
3	9	4	7	4	13	29	14	5	23	108
4	9	3	5	2	6	18	8	11	16	78
5	6	5	6	2	6	24	9	3	17	78
6	5	2	3	3	1	16	5	1	9	45
7	2	.....	4	3	4	11	12	3	11	50
8	2	1	2	.....	2	13	4	1	8	33
9	.....	1	1	1	3	4	5	3	5	23
10	1	.....	1	.....	2	8	5	.....	4	21
11	1	1	.....	.....	2	2	4	1	1	12
12	.....	2	1	.....	1	2	.....	.....	2	8
13	.....	.....	2	.....	1	4	.....	1	2	10
14	.....	.....	1	.....	1	.....	1	1	.....	4
15	.....	.....	1	.....	1	2	1	.....	.....	5
16	1	2	.....	.....	2	.....	1	.....	.....	6
17	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
18	.....	.....	.....	.....	.....	2	1	.....	.....	3
19	.....	.....	2	.....	.....	.....	.....	.....	.....	2
20	.....	.....	1	.....	.....	1	.....	.....	.....	2
21	.....	.....	.....	.....	.....	1	.....	.....	.....	1
22	.....	.....	.....	.....	1	.....	.....	.....	.....	1
23	.....	.....	1	.....	.....	.....	.....	.....	.....	1
24	.....	.....	.....	.....	.....	.....	.....	.....	1	1
25	.....	.....	1	.....	.....	.....	.....	.....	.....	1
26	.....	.....	1	.....	.....	.....	.....	.....	.....	1
27	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
28	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
29	.....	.....	.....	.....	.....	.....	1	.....	.....	1
30	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
31	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
32	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
33	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
34	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
35	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
36	.....	.....	.....	.....	.....	.....	.....	.....	.....	0
37	.....	.....	.....	.....	.....	.....	1	.....	.....	1
Totals.....	88	44	75	68	75	328	149	73	211	1111
Medians.....	1.5	2.9	2.4	1.6	3.7	2.4	2.8	1.8	2.7	2.6
25 percentile.	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.9
75 percentile.	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.4
Quartile.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	2.3

TABLE XXXVII.—*Number of pupils receiving each negative grade for Test E.*

Score	I	II	III	IV	V	VI	VII	LV	Totals
0	18	20	76	9	28	10	10	7	178
1	14	19	117	6	21	19	9	10	215
2	33	14	147	8	14	23	4	3	246
3	17	13	92	4	16	18	2	6	168
4	15	7	56	6	4	7	4	6	105
5	8	2	24	3	4	10	.....	5	56
6	3	1	16	1	4	3	1	3	32
7	5	1	6	1	1	3	.....	2	19
8	.....	.....	2	2	.....	3	.....	.....	7
9	.....	.....	2	.....	.....	.....	.....	1	3
10	1	.....	2	.....	.....	.....	.....	1	4
11	.....	.....	.....	.....	.....	.....	.....	.....	0
12	.....	.....	1	.....	.....	.....	.....	.....	1
13	.....	.....	1	.....	.....	1	.....	.....	2
Totals.....	114	77	542	40	92	97	30	44	1036
Medians.....	2.8	2.0	2.5	2.6	1.9	2.8	1.6	3.3	2.5
25 percentile ..	.....	.....	.....	.....	.....	.....	.....	.....	1.4
75 percentile ..	.....	.....	.....	.....	.....	.....	.....	.....	3.8
Quartile.....	.....	.....	.....	.....	.....	.....	.....	.....	1.2

Conclusions.—Summaries of the results in Tables XXVII to XXXVII are given in Tables XXXVIII and XXXIX. Since, as previously noted, we do not know that the tests measure the respective abilities in the same way, we can not compare the results of the different tests. Thus the low median positive score for Test C does not necessarily indicate that the pupils have less ability to recall facts about a figure than to do any one of the things called for in the other tests.[1] However we may compare results obtained by giving the same test in different schools. Such a comparison shows a decided variation in both the positive and negative scores made by the schools taking any one of the tests. In the case of each test, the marks of some schools are quite satisfactory while those of others are extremely low. This variation in achievement[2] may be due, in part, to

[1] The low median scores for Test C are, in part, due to the arbitrary selection of the number of facts required for a perfect answer to each exercise.

[2] The high maximum score for Test E may be due to the fact that the last exercise of the test had been studied in class. The test papers indicate that this might be the case although the evidence is not conclusive.

TABLE XXXVIII.—*Summary of positive scores.*

Test	Medians for All Pupils Tested	25 Percentile	75 Percentile	Lowest Median by Any School	Highest Median by Any School
A	62.5	51.3	72.9	50.5	78.7
B	69.3	51.8	82.2	38.5	80.9
C	50.6	36.5	65.2	29.0	67.0
D	73.3	55.4	87.0	54.7	80.5
E	61.5	28.7	67.5	28.7	99.0

TABLE XXXIX.—*Summary of negative scores.*

Test	Median for All Pupils Tested	25 Percentile	75 Percentile	Lowest[1] Median by Any School	Highest Median by Any School
A	7.1	10.7	4.2	11.8	4.8
B	3.5	5.9	1.5	4.5	2.0
C	4.1	7.3	1.9	7.3	2.4
D	2.6	5.4	0.9	3.7	1.5
E	2.5	3.8	1.4	3.3	1.6

differences in local conditions rather than differences in methods and in teaching ability. Nevertheless, it is difficult to see how local conditions alone could result in the extremely low scores made by some of the schools. If the abilities tested are essential to success in the study of geometry, then the results indicate that progress is almost impossible in some of the schools until these abilities have been further developed. On the other hand the achievements of other schools indicate that it is altogether possible to develop these abilities to a fair degree during the study of the first two books of geometry.

School LXII, which had completed plane geometry took Test A and made a positive score of 71.5 and a negative score of 4.6. While this school ranks high it did not make a better showing than some of the schools which had completed the first two books only. This again raises the question of the effect of further training such as is now given in our schools.

X. USE OF THE TESTS

Thus, although it is possible to develop the abilities with which this study is concerned, some schools fail to do so. Therefore, if these abilities are essential to progress in geometry, it is important that we have some means of determining whether they are being satisfactorily developed in a class. Such a

[1] As the negative scores represent the numbers of incorrect and unnecessary statements, the larger numbers represent the lower scores.

diagnosis will enable the teacher to give attention to the particular phases of the subject in which the pupils are weak. It is believed that Tests A, B, C and D may be found useful for this purpose. As nearly as possible, each of these tests has been arranged to measure a single ability. Moreover, the method of grading reveals to the teacher not only the pupils' *positive* abilities but also the extent to which they are influenced by misconceptions. These analytic features of the tests are of importance, for it is only by determining the elements of the pupils' abilities that we may know where to place the emphasis in our teaching.

The standards given in Tables XXXVIII and XXXIX furnish a means of comparison. If a class is above the median scores in these tables, and especially if it is near or above the 75-percentile marks, the teacher may be fairly sure that the abilities have been sufficiently developed to insure the success of the class. If, however, the class falls below the median score in any one of the tests and especially if it falls near or below the 25-percentile mark, special effort should be made to develop the ability in question.

If such comparisons are to be trustworthy, the tests should be given at the time the class has completed the first two books of geometry, and the rules for scoring the tests given on pages 97–99 of the appendix[1] should be followed carefully. Furthermore, it should be remembered that these tests do not cover the entire field of geometry. They deal with abilities essential to the formal demonstration of theorems; but there are other phases of the subject, such as the practical, which we have not investigated and it is possible that a class may make a creditable showing on each of these tests and yet not realize the greatest values from geometry. Hence teachers should not rely wholly upon these tests as a means of determining the weaknesses of their classes.

XI. Conclusions

This concludes the more important features of our study. Some minor and related topics will be discussed in the appendix. We have assumed that the abilities investigated are essential to the study of geometry. This assumption is based upon long experience as a teacher and upon practice in our schools all of which

[1] See also the fuller discussion of the method of scoring papers on pages 28–43.

emphasize these abilities in some way or other. In some cases the school grades bear a close relation to these abilities, but usually this relation is slight; so slight, in fact, that if it were a true index of the value of these abilities the time spent on their development could not be justified. However, this condition is perhaps due, in part, to the teachers' inability to grade their pupils accurately. In like manner, when judged by the scores made on any one of the tests, the schools vary greatly in their achievements. While the returns from some of the schools are fairly satisfactory, in many cases the scores are so low as to make it doubtful whether values dependent upon these abilities are realized. This variation in achievement may be due, in part, to local conditions; but it is doubtless dependent, to a certain extent, upon the teachers' efficiency, which, we believe, could be increased if tests similar to these were used to show where emphasis should be placed.

APPENDIX

For the benefit of any who care to give the tests and compare their results with those of this investigation a brief statement of directions for scoring the papers is given below. Also certain inquiries made by teachers have been embodied in an Information Blank which was sent to each school giving the tests. This blank was returned by all except schools IV, XII, XIII and XXII. The data thus gathered is included in this appendix.

I. A Brief Statement of Directions for Scoring Papers[1]

Test A.—1. The positive values assigned to exercises I, II, III, IV and V, of Test A are 15, 17, 19, 23 and 26 respectively. The necessary steps for a perfect answer to each exercise are given on pages 28–31. The pupil's positive score is obtained by marking each exercise on the basis of the value assigned to it and taking the sum of such marks.

2. The negative score is the total number of incorrect and unnecessary drawings in his paper.

3. If a line or a part of a figure ought to fulfill two or more conditions but the pupil has drawn it to fulfill only a part of these conditions, credit is given for the correct points and the incorrect points are counted in the negative score.

4. Any unnecessary drawings are to be counted in the negative score. In particular, if in exercise IV, the pupil draws the bisectors of the interior and exterior vertical angles at each of the three vertices, the drawings at two of them are counted as unnecessary.

5. The lettering of figures is not considered when scoring the papers unless A and B are incorrectly used in exercise I.

6. If a pupil draws a special figure for any one of the exercises but draws it correctly, full credit is given.

7. If in exercise III, the medians are not produced to the midpoints of the opposite side but would pass through such points if produced, full credit is given for the drawing.

[1] For a fuller discussion of the directions for scoring see pages 28–43.

Test B.—1. The positive values assigned to exercises I, II, III and IV of Test B are 21, 23, 26 and 30 respectively. The necessary steps for a perfect answer to each exercise are given on pages 32–34. The pupil's positive score is determined by marking the hypothesis and conclusion of each exercise separately on the basis of the value assigned to it and then averaging these two marks. The sum of these averages for all the exercises of the test is the pupil's final positive score.

2. The negative score is the total number of incorrect and unnecessary statements in the entire test.

3. A statement is counted as correct only when it is given correctly in terms of the figure, but care must be taken not to count off twice for the same lack of specific statement.

4. General statements not given in terms of the figure are not counted in determining the negative score unless such statements are given incorrectly.

5. Credit is not given for parts of the hypothesis stated in the conclusion excepted as noted in 6 below.

6. If in exercise IV the pupil has included $CD = \frac{1}{2}AB$, $CD > \frac{1}{2}AB$, $CD < \frac{1}{2}AB$ correctly as conditional clauses in the conclusion, full credit for each statement as a part of the hypothesis is given.

Test C.—1. The positive values assigned to exercises I, II, III and IV of Test C are 25, 24, 27 and 24 respectively, and the numbers of correct statements considered as perfect answers are 8, 30, 7 and 18 respectively. The pupil's positive grade is obtained by marking each exercise on the basis of the value assigned to it and taking the sum of these marks for all the exercises of the test.

2. The negative score is the sum of all incorrect statements in the entire test.

3. A statement is counted as correct only when it gives a geometrical relation or a magnitude correctly in terms of the figure.

4. General statements not given in terms of the figure are not counted in determining the negative score unless they are incorrectly stated.

5. Full credit is given for all facts included in a continued equation or inequality, but double credit is not given for statements repeated in the same or slightly different forms.

6. If a pupil makes additional drawings all statements involving such drawings are to be eliminated.

Test D.—1. The positive values assigned to exercises I, II and III of Test D are 27, 34 and 39 respectively. The number of steps in a correct answer depends upon the method of proof used by the pupil. These steps are given on pages 38–41 for all proofs found in the papers graded by the author. The pupil's positive score is obtained by marking each exercise on the basis of the value assigned to it and adding these marks for all exercises of the test.

2. The negative score is the sum of all incorrect and unnecessary statements in the entire test.

3. Any reasons or authorities for the various steps in the proof are to be disregarded.

4. If a statement is out of its logical order it is counted as incorrect unless its relation is indicated in some way (*e. g.*, by "for," "since," etc.).

5. If a proof is incomplete, the pupil should receive credit for the number of correct steps given. If it is possible to complete the pupil's proof in more than one way, the smallest possible number of steps is taken as the required number for a perfect answer.

Test E.—1. The positive values assigned to exercises I, II, III and IV of Test E are 12, 16, 33 and 39 respectively. If the drawings made by a pupil for an exercise make a proof possible, the positive score is the full value assigned to that exercise. Otherwise it is zero. The sum of the marks for all exercises of the test is the pupil's final positive score.

2. The negative score is the sum of all incorrect and unnecessary drawings in the entire test.

3. Any drawing which leads to a proof is counted as correct. Any additional drawings are considered as unnecessary.

II. Information Blank

The following is a copy of the information blank which each school was requested to fill out and return to the author.

Information Blank

The head of the Mathematical Department will please give the following information concerning the classes tested:

1. What text in geometry was used?.....
2. How many weeks were given to the first two books of geometry?.....
3. How many recitations per week were given to geometry?.....
4. In which year of the high school course were the first two books of geometry studied?.....
5. How much algebra did the pupils have before beginning the course in formal geometry?.....
6. Had the pupils had a preliminary course in
   a. Experimental geometry?.....
   b. Constructional geometry?.....
7. How long were these preliminary courses?.....
8. When were these preliminary courses given?.....
9. State briefly the method of instruction used. (Especially any features of the method which would affect the results of the test given.).....
.....
.....
10. On the reverse side of this sheet give any additional facts which you think would influence the results of the tests.
11. Name of school.....
12. Name of person giving this information.....

III. The Text Book

Each of the following text books was used by one or more of the schools giving the tests:

1. Durell
2. Durell and Arnold
3. Ford and Ammerman
4. Hart and Feldman
5. Lyman
6. Milne
7. Palmer and Taylor
8. Phillip and Fisher
9. Robbins
10. Schultze and Sevenoak
11. Stone-Millis
12. Wentworth
13. Wentworth and Smith
14. Wells
15. Wells and Hart

Table XL indicates the text book used by each school and also the test which was given in that school. The numbers refer to the books given above. Thus, school VI in which Test E was given used two books; namely, Durell and Wells. The data of this table has no scientific value and no conclusions should be drawn from it. It only answers the question so often asked by teachers who gave the tests, "What text is used by a given school?"

TABLE XL.—*Text book used by each school.*

Test A		Test B		Test C		Test D		Test E	
School	Text	School	Text	School	Text	School	Text	School	Text
XXIII	13	XIV	13	VII	13	V	11	I	4
XXV	1	XV	13	VIII	10	XXIII	13	II	10
XXXIII	13	XVI	13	IX	13	XXIV	13	III	10
XXXV	13	XVII	13	X	13	XXV	1	V	11
XXXVI	13	XVIII	13	XI	13	XXVI	4	VI	1, 14
XXXIX	4	XIX	4	XXVIII	13	XXVII	9	VII	13
XL	13	XX	10	XXXII	7	XXIX	8	LV	13
XLI	15	XXI	13	XXXIV	13	XLVI	13		
XLII	6	XXV	1	XXXVII	13				
XLIII	9	XXX	13	XXXVIII	3				
XLIV	15	XXXI	4						
XLV	2	XLVII	13						
L	13	XLVIII	2						
LI	13	XLIX	2						
LII	13	LVII	13						
LIII	5, 12	LVIII	13						
LIV	1								
LVI	13								
LIX	4, 8								
LX	2								
LXI	13								
LXII	5								
LXIII	1								

IV. THE AMOUNT OF TIME GIVEN TO THE FIRST TWO BOOKS OF GEOMETRY

The number of weeks given to the first two books of geometry varies from 11 to 33. Most schools devote five recitations per week to geometry, while some devote only four, and a very few

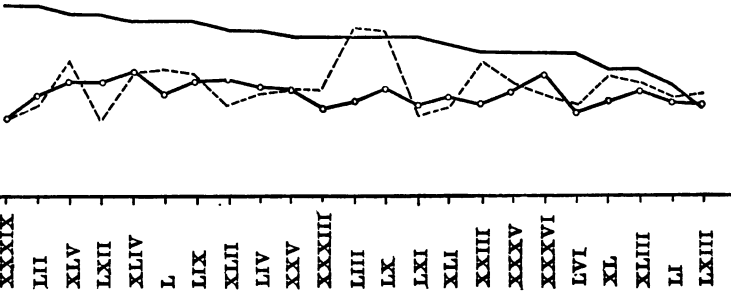


FIG. 24. Relation of time devoted to the first two books of geometry to the median scores made on Test A.

——— = Number of recitations devoted to the first two books.

o–o–o–o = Median positive score made by each school.

– – – – – = Median negative score made by each school.

TABLE XLI.—*Number of recitations given to the first two books of geometry.*

Test A		Test B		Test C		Test D		Test E	
School	No. Rec.	School	No. Rec.	School	No. Rec.	School	No. Rec.	School	No. Rec.
XXIII	90	XIV	80	VII	80	V	90	I	100
XXV	100	XV	100	VIII	100	XXIII	90	II	120
XXXIII	100	XVI	100	IX	92	XXIV	90	III	100
XXXV	90	XVII	100	X	80	XXV	100	V	90
XXXVI	90	XVIII	105	XI	100	XXVI	95	VI	100
XXXIX	120	XIX	70	XXVIII	115	XXVII	95	VII	80
XL	80	XX	105	XXXII	48	XXIX	80	LV	100
XLI	95	XXI	105	XXXIV	80	XLVI	110		
XLII	105	XXV	100	XXXVII	90				
XLIII	80	XXX	85	XXXVIII	105				
XLIV	110	XXXI	80						
XLV	115	XLVII	120						
L	110	XLVIII	125						
LI	70	XLIX	65						
LII	120	LVII	110						
LIII	100	LVIII	150						
LIV	105								
LVI	90								
LIX	110								
LX	100								
LXI	100								
LXII	115								
LXIII	55								

only three recitations to the subject. Table XLI indicates the total number of recitations given to the first two books of geometry by each school. In the case of schools XVIII and XIX the indefinite way in which the data were given made it impossible to do more than give approximate results.

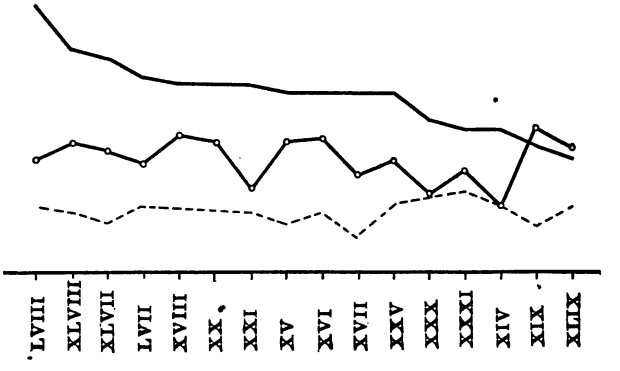


FIG. 25. Relation of time devoted to the first two books of geometry to the median scores made on Test B.

The number of periods devoted to the first two books of geometry, the positive scores, and the negative scores for Tests A, B, C, D and E are represented graphically in Figs. 24 to 28 respectively. If there were a positive correlation between the

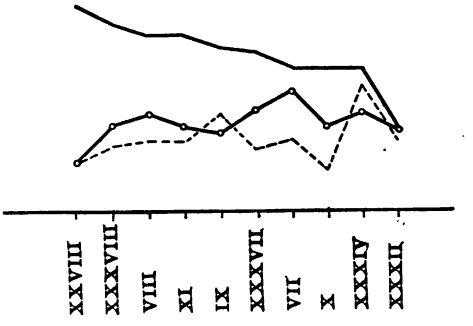


FIG. 26. Relation of the time devoted to the first two books of geometry to the median scores made on Test C.

number of recitations and the test scores, then the curve for the positive scores would fall and that for the negative scores would rise as the curve for the number of recitations falls. Figure 24 shows that there is but slight, if any, relation between the scores

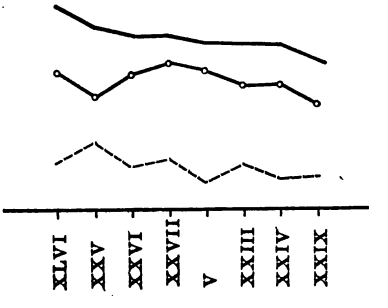


FIG. 27. Relation of time devoted to the first two books of geometry to the median scores made on Test D.

for Test A and the number of recitations. Figure 25 shows a similar condition to exist for the positive scores for Test B, while there is, perhaps, a slight positive correlation between the negative scores and the number of recitations. From Fig. 26 it appears that the larger the number of recitations given to the first two books of geometry the fewer are the facts which the

pupils are able to recall. However, the number of errors tends to decrease as the number of recitations increases. In the case of Test D (Fig. 27) the number of recitations seems to bear a slight positive relation to the positive scores and a slight negative relation to the negative scores. In Fig. 28 it is difficult to detect any relation between the test scores and the number of recitations.

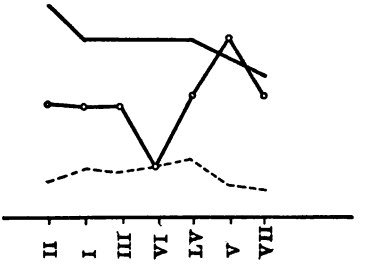


FIG. 28. Relation of time devoted to the first two books of geometry to the median scores made on Test E.

It must be remembered that the number of schools tested is too small and the conditions in these schools are too varied to permit of any definite conclusions being drawn. However, these data do raise a question as to whether excessive time spent on the first two books of geometry is justifiable in so far as the abilities tested are concerned.

V. THE PLACE OF GEOMETRY IN THE CURRICULUM

Table XLII indicates the year in which the first two books of geometry are given in each of the schools returning the Information Blank. For the purpose of comparison the median scores are also given. A large majority of the schools give the first two books of geometry some time during the second year of high school. In fact there is so little variation from this that we can draw no conclusions as to what is the best time to begin the study of formal geometry. However, it is noteworthy that school V made among the highest scores although it began the study of geometry during the first year of high school, and that no school which began the study in the third year made an exceptionally high score. This raises (but does not answer) a question as to whether it is best to put off the study of plane geometry until the more advanced years of the high school.

TABLE XLII.—*The school year in which each school gives the first two books of geometry.*

Test A				Test B				Test C				Test D				Test E			
Schools	Year	Positive Score	Negative Score	Schools	Year	Positive Score	Negative Score	Schools	Year	Positive Score	Negative Score	Schools	Year	Positive Score	Negative Score	Schools	Year	Positive Score	Negative Score
XXIII	2	59.0	8.5	XIV	3	38.5	3.8	VII	2	67.0	4.0	V	1	75.0	1.5	I	2	61.6	2.8
XXV	2	57.7	6.9	XV	2	74.5	2.8	VIII	2	54.1	4.0	XXIII	2	71.5	2.4	II	2	62.0	2.0
XXXIII	2	54.8	6.6	XVI	2	74.5	3.2	IX	2	44.4	5.3	XXIV	2	68.8	1.6	III	1	61.4	2.5
XXXV	2	66.5	7.1	XVII	2	54.0	2.0	X	2	46.3	2.4	XXV	2	61.6	3.7	V	1	99.0	1.9
XXXVI	2	57.0	6.5	XVIII	2	56.2	3.5	XI	2	49.5	4.0	XXVI	2	74.0	2.4	VI	3	28.7	2.8
XXXIX	2	50.5	4.8	XIX	2	80.9	2.6	XXVIII	2	29.0	3.0	XXVII	2	80.5	2.8	VII	2	67.2	1.6
XL	2	60.5	7.9	XX	2	72.7	3.5	XXXII	3	45.0	4.0	XXIX	2	54.7	1.8	LV	3	67.3	3.3
XLI	2	62.5	5.7	XXI	2	47.5	3.4	XXXIV	2	55.2	7.3	XLVI	2	74.7	2.7				
XLII	2	72.5	5.6	XXV	2	61.5	3.8	XXXVII	3	56.0	3.5								
XLIII	2	68.5	7.4	XXX	2	43.3	4.3	XXXVIII	2	49.0	3.6								
XLIV	2	78.7	7.8	XXXI	3	56.8	4.5												
XLV	2	72.5	8.5	XLVII	2	69.0	2.8												
L	3	64.0	8.0	XLVIII	2	71.8	3.4												
LI	3	60.0	6.3	XLIX	2	70.5	3.6												
LII	2	62.0	5.7	LVII	2	61.0	3.8												
LIII	3	59.8	11.8	LVIII	2	63.5	3.8												
LIV	2	69.0	6.5																
LVI	2	52.8	5.8																
LIX	2	72.2	7.6																
LX	3	68.3	11.4																
LXI	2	57.8	5.1																
LXII	2	71.5	4.6																
LXIII	2	58.3	6.6																

VI. The Amount of Time Devoted to Algebra before Beginning the Study of Plane Geometry

In answer to question 5 of the Information Blank most schools stated the amount of time devoted to algebra before the study of geometry was begun. A few schools stated the amount of work done. In order to make the data comparable we have, in the latter cases, replaced the amount of work done by the time usually required to do that work, although we realize that schools vary as to the amount of time devoted to a given amount of work. The data from question 5 together with the median scores are given in Table XLIII. A study of this table shows that an increased length of time given to the study of algebra does not necessarily mean an increase in ability to do these tests. This result is to be expected, for the abilities investigated in this study have but little or no relation to algebra as now taught in most of our high schools.

VII. Experimental and Constructional Geometry

Only five schools reported that anything in the way of experimental or constructional geometry had been done before the pupils began their study of formal geometry. The pupils of schools III and LIX had a half year of constructional geometry during the first year of high school, and school VI gave three recitations per week during the second half of the second year. In school XVIII some constructional work was given in connection with the algebra of the first year, and school XLIX devoted the first four weeks of the second year to constructional geometry.

VIII. The Methods Used

The answers to question 9 were so varied that it was impossible to classify them except in a very rough way. Twenty-one of the schools returning the Information Blank did not answer this question. Twenty-eight schools discussed the class management rather than methods, while the answers of twenty-seven contained statements relative to the content of the course. Six schools indicated their methods by such general terms as syllabus, synthetic, inductive, analytic and heuristic. Only six of the schools indicated that their methods were directly intended to

TABLE XLIII.—*Time given to the study of algebra before beginning the study of geometry.*

Test A				Test B				Test C				Test D				Test E			
Schools	Time in Years	Positive Score	Negative Score	Schools	Time in Years	Positive Score	Negative Score	Schools	Time in Years	Positive Score	Negative Score	Schools	Time in Years	Positive Score	Negative Score	Schools	Time in Years	Positive Score	Negative Score
XXIII	0.00	59.0	8.5	XIV	1.00	38.5	3.8	VII	1.50	67.0	4.0	V	1.00	75.0	1.5	I	1.00	61.6	2.8
XXV	1.00	57.7	6.9	XV	1.00	74.5	2.8	VIII	1.00	54.1	4.0	XXIII	0.00	71.5	2.4	II	1.00	62.0	2.0
XXXIII	1.00	54.8	6.6	XVI	1.50	74.5	3.2	IX	1.00	44.4	5.3	XXIV	1.00	68.8	1.6	III	0.50	61.4	2.5
XXXV	1.50	66.5	7.1	XVII	1.50	54.0	2.0	X	1.50	46.3	2.4	XXV	1.00	61.6	3.7	V	1.00	99.0	1.9
XXXVI	1.00	57.0	6.5	XVIII	1.00	76.2	3.5	XI	1.50	49.5	4.0	XXVI	1.00	74.0	2.4	VI	1.50	28.7	2.8
XXXIX	1.00	50.5	4.8	XIX	1.50	80.9	2.6	XXVIII	1.75	29.0	3.0	XXVII	1.00	80.5	2.8	VII	1.50	67.2	1.6
XL	1.00	60.5	7.9	XX	1.00	72.7	3.5	XXXII	1.00	45.0	4.0	XXIX	1.00	54.7	1.8	LV	2.00	67.3	3.3
XLI	1.00	62.5	5.7	XXI	1.00	47.5	3.4	XXXIV	1.00	55.2	7.3	XLVI	1.00	74.7	2.7				
XLII	1.00	72.5	5.6	XXV	1.00	61.5	3.8	XXXVII	2.00	56.0	3.5								
XLIII	1.00	68.5	7.4	XXX	1.00	43.3	4.3	XXXVIII	1.00	49.0	3.6								
XLIV	1.00	78.7	7.8	XXXI	1.00	56.8	4.5												
XLV	1.00	72.5	8.5	XLVII	1.00	69.0	2.8												
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LIX	1.00	72.2	7.6																
LX	1.50	68.3	11.4																
LXI	2.00	57.8	5.1																
LXII	1.00	71.5	4.6																
LXIII	1.00	58.3	6.6																

develop the abilities with which this study is concerned. In fact, if we may judge from the answers to question 9, it is doubtful if the methods of many teachers are suited to the development of these abilities.


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